Enhancement of commensurate magnetic resonance energy by the additional second neighbor hopping $t'$ in cuprate superconductors

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Abstract

Within the $t$–$t'$–$J$ model, the effect of the additional second neighbor hopping $t'$ on the spin response of doped cuprates in the superconducting-state is discussed under the kinetic energy driven superconducting mechanism. It is shown that the additional second neighbor hopping $t'$ is systematically accompanied with the enhancement of the commensurate magnetic resonance energy, and therefore it plays an important role in quantitatively determining the commensurate magnetic resonance energy. The incommensurate magnetic scattering peaks are found at $[\{1 \pm \delta\} \pi, \pi]$ and $[\pi, \{1 \pm \delta\} \pi]$ at low energy and $[\{1 \pm \delta'\} \pi, \{1 \pm \delta'\} \pi]$ at high energy, with the magnetic excitations at high energy disperse almost linearly with energy, and have energies greater than the superconducting gap energy, and are present at the superconducting transition temperature. These incommensurate magnetic scattering peaks from both low and high energies converge to the commensurate $[\pi, \pi]$ magnetic resonance peak at intermediate energy, with the commensurate magnetic resonance energy increases with increasing doping in the underdoped regime, and reaches a maximum in the optimal doping, then decreases in the overdoped regime.

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The interplay between the antiferromagnetic (AF) short-range correlation (AFSRC) and superconductivity is one of the challenging issues for the theory of cuprate superconductors [1–3]. The undoped cuprates are insulating antiferromagnets with the AF long-range-order (AFLRO) [4,5]. Although this AFLRO reduced dramatically with the charge carrier doping and vanishes around the doping 5%, AFSRC still persists even in the superconducting (SC)-state, therefore there is a coexistence of AFSRC and superconductivity [4,5]. Experimentally, by virtue of systematic studies using the nuclear magnetic resonance, and muon spin rotation techniques, particularly the inelastic neutron scattering, the doping and energy dependent spin response in doped cuprates in the SC-state have been well established now. At low energy, the incommensurate (IC) magnetic scattering peaks are shifted from the AF wave vector $[\pi, \pi]$ to four points $[(1 \pm \delta) \pi, \pi]$ and $[\pi, (1 \pm \delta) \pi]$ (in units of inverse lattice constant) with $\delta$ as the IC parameter [6–8]. However, with increasing energy these IC magnetic scattering peaks converge on the commensurate $[\pi, \pi]$ magnetic resonance peak at intermediate energy [7,9–11], and then well above this magnetic resonance energy, the continuum of magnetic excitations peaked at the IC positions $[(1 \pm \delta') \pi, (1 \pm \delta') \pi]$ in the diagonal direction are observed [12,13]. In particular, the geometry of these IC magnetic excitations is two-dimensional [12,14]. Although some of these magnetic properties have been observed in the normal-state, these IC magnetic scattering and commensurate $[\pi, \pi]$ resonance are the main new feature that appears into the SC-state [6–13]. Moreover, these unusual magnetic excitations at high energy have energies greater than the SC pairing energy, are present at the SC transition temperature, and have spectral weight far exceeding that of the resonance [12].

Recently, we [15] have developed a charge-spin separation (CSS) fermion-spin theory for description of the physical properties of cuprate superconductors, where the electron operator is decoupled as a gauge invariant dressed holon and spin. Within this
theoretical framework, we have discussed the unusual normal-state properties and kinetic energy driven SC mechanism [15,16]. It is shown that the charge transport is mainly governed by the scattering from the dressed holons due to the dressed spin fluctuation, while the scattering from the dressed spins due to the dressed holon fluctuation dominates the spin response [15]. Based on the $t$–$J$ model, it is also shown [16] that the dressed holons interact occurring directly through the kinetic energy by exchanging the spin excitations, leading to a net attractive force between the dressed holons, then the electron Cooper pairs originating from the dressed holon pairing state are due to the charge-spin recombination, and their condensation reveals the SC ground-state. This SC-state is controlled by both SC gap function and quasiparticle coherence, and the maximal SC transition temperature occurs around the optimal doping, then decreases in both underdoped and overdoped regimes [17]. Moreover, AFSRC has been incorporated into the electron Cooper pair in terms of the charge-spin recombination, therefore there is a coexistence of the electron Cooper pair and AFSRC, and then AFSRC can persist into superconductivity [16]. Although the $t$–$J$ model captures the essential physics of cuprate superconductors [1], it has been shown from the angle resolved photoemission spectroscopy (ARPES) experiments [5,18] that the quantitative comparison with cuprate superconductors may be properly accounted by generalizing the $t$–$J$ model to include the second- and third-nearest neighbors hopping terms $t'$ and $t''$. Moreover, the experimental analysis [19] shows that some differences among the different families of cuprate superconductors is strongly correlated with $t'$. Since both experiments from ARPES and neutron scattering measurements produce interesting data that introduce important constraints on the microscopic models and SC theories for cuprate superconductors, then a natural question is what is the effect of these additional hoppings on the doping and energy dependent spin response in the SC-state. In this Letter, we discuss this issue under the framework of the kinetic energy driven SC mechanism [16]. Within the $t$–$t'$–$J$ model, we have performed a systematic calculation for the dynamical spin structure factors in the SC-state, and quantitatively reproduced some main features found in the experiments on cuprate superconductors, including the doping and energy dependence of the IC magnetic scattering at both low and high energies [11–13] and commensurate $[\pi, \pi]$ resonance at intermediate energy [9,10]. Our result also shows that the additional second neighbor hopping $t'$ is systematically accompanied with the enhancement of the magnetic resonance energy, and therefore it plays an important role in quantitatively determining the magnetic resonance energy.

In order to account for the real experiments for the $t$–$t'$–$J$ model, the crucial requirement is to impose the electron local constraint $\sum_\sigma C^\dagger_\sigma C_\sigma \leq 1$ to avoid the double occupancy [1,2]. It has been shown that this local constraint can be treated properly in analytical calculations within the CSS fermion-spin theory [15], where the constrained electron operators are decoupled as $C^\dagger_\sigma = h^\dagger_i \sigma S^\dagger_\sigma$ and $C_i = h^+_i \sigma S^+_\sigma$, with the spinful fermion operator $h_\sigma = e^{-i\Phi_\sigma} \hat{h}_i$ describes the charge degree of freedom together with some effects of the spin configuration rearrangements due to the presence of the doped hole itself (dressed holon), while the spin operator $S^\dagger_\sigma$ describes the spin degree of freedom (dressed spin), then the electron local constraint for the single occupancy, $\sum_\sigma C^\dagger_\sigma C_\sigma = S^+_\uparrow h_\uparrow h^+_\uparrow S_\uparrow + S^-_\downarrow h_\downarrow h^+_\downarrow S^-_\downarrow + h_\uparrow h^+_\uparrow S^-_\downarrow + h_\downarrow h^+_\downarrow S^+_\uparrow + 1 = h_\uparrow^2 \leq 1$, is satisfied in analytical calculations. It has been also shown that these dressed holon and spin are gauge invariant [15], and in this sense, they are real and can be interpreted as the physical excitations [20]. Although in common sense $h_\sigma$ is not a real spinful fermion, it behaves like a spinful fermion. In this CSS fermion-spin representation, the low-energy behavior of the $t$–$t'$–$J$ model on a square lattice can be expressed as [15],

\[
H = -t \sum_{i\tilde{\eta}} (h^\dagger_i S^\dagger_{i+\tilde{\eta}} h^+_i S^-_i + h_i S^-_i h^+_i S^\dagger_{i+\tilde{\eta}} + t' \sum_{i\tilde{\xi}} (h^\dagger_i S^\dagger_{i+\tilde{\xi}} h^+_i S^-_{i+\tilde{\xi}} + h_i S^-_i h^+_i S^\dagger_{i+\tilde{\xi}}) + \mu \sum_{i\sigma} \hat{h}^\dagger_i h_i \sigma + J_{\text{eff}} \sum_{i\tilde{\eta}} S_i \cdot S_{i+\tilde{\eta}},
\]

where $\hat{\eta} = \pm \tilde{\xi}$, $\pm \tilde{\eta}$, $\tilde{\eta} = \pm \tilde{\xi} \pm \tilde{\eta}$, $S_\sigma = C^\dagger_\sigma \hat{C}_\sigma / 2$ is spin operator with $\hat{C}_\sigma = (\sigma_x, \sigma_y, \sigma_z)$ as Pauli matrices, $\mu$ is the chemical potential, $J_{\text{eff}} = (1 - x)^2 J$, and $x = (h^\dagger_i h_i \sigma) = (h_i^+ h_i) / 2$ is the hole doping concentration. As a consequence, the kinetic energy terms in the $t$–$t'$–$J$ model have been expressed as the dressed holon–spin interactions, which reflects that even the kinetic energy terms in the $t$–$t'$–$J$ Hamiltonian have strong Coulombic contributions due to the restriction of no doubly occupancy of a given site, and therefore dominate the essential physics of cuprate superconductors.

In the CSS fermion-spin theory, the magnetic fluctuation is dominated by the scattering of the dressed spins [15,21]. In the normal-state, the dressed spins move in the dressed holon background, then the dressed spin self-energy (then full dressed spin Green’s function) in the normal-state has been obtained [15,21] within the framework of the equation of motion method [22,23] in terms of the collective mode in the dressed holon particle–hole channel. With the help of this full dressed spin Green’s function in the normal-state, the IC magnetic scattering and integrated spin response of doped cuprates in the normal-state have been discussed [15,21], and the results of the doping dependence of the incommensurability and integrated dynamical spin susceptibility are consistent with the experimental results in the normal-state [4,6,7]. However, in the framework of the kinetic energy driven superconductivity [16], the SC-state is originated from the dressed holon pairing state, and has the $d$-wave symmetry in a wide range of doping [17], therefore in the SC-state the dressed spins move in the dressed holon pair background. In this case, the doping and energy dependent spin response of the $t$–$J$ model in the SC-state with the $d$-wave symmetry has been discussed in terms of the collective mode in the dressed holon particle–particle channel [17]. Following our previous discussions for the $t$–$J$ model case [17],
the full dressed spin Green’s function in the present $t' \rightarrow J$ model in the SC-state is obtained as,
\[
D(k, i\omega_n) = \frac{1}{D^{(0)-1}(k, i\omega_n) - \Sigma^{(s)}(k, i\omega_n)},
\]
with the dressed spin mean-field (MF) Green’s function [15,16],
\[
D^{(0)}(k, i\omega_n) = B_k \left( \frac{1}{i\omega_n - \omega_k} - \frac{1}{i\omega_n + \omega_k} \right),
\]
where $B_k = 2\lambda_1 (A_1 \gamma_k - A_2) - \lambda_2 (2\chi_1' \gamma_k - \chi_2)$, $\lambda_1 = 2ZJ_{\text{eff}}$, $\lambda_2 = 4Z\phi_2 t'$, $A_1 = \epsilon \chi_1' + \chi_2 / 2$, $A_2 = \chi_1' + \epsilon \chi_1 / 2$, $\epsilon = 1 + 2t\phi_1 / J_{\text{eff}}$, $\gamma_k = (1/Z) \sum_{\vec{q}} e^{i\vec{q} \cdot \vec{r}} \gamma_k' \gamma_{\vec{q}} = (1/Z) \sum_{\vec{r}} e^{i\vec{r} \cdot \vec{t}} \gamma$, $Z$ is the number of the nearest-neighbor or next-nearest-neighbor sites, the dressed holon’s particle–hole parameters $\phi_1 = (h_{1\sigma} h_{1+\sigma})$ and $\phi_2 = (h_{1\sigma} h_{1+\sigma})$, the spin correlation functions $\chi_k = (S_1^z S_{-1+\sigma}^z)$, $\chi_{\vec{r}} = (S_{1+\vec{r}}^z S_{-1+\vec{r}}^z)$, and $\chi_2 = (S_{1+\vec{r}}^z S_{-1+\vec{r}}^z)$, and the MF dressed spin excitation spectrum,
\[
\omega_k^2 = \lambda_1^2 \chi_k \left[ 1 + \frac{1}{2Z} \left( \alpha \chi_1' \gamma_k \right) + \frac{1}{2} \left( \alpha A \gamma_k \right) \right] + \lambda_2^2 \left( \alpha \chi_1' \gamma_k + \frac{1}{2} \left( \alpha A \gamma_k \right) \right) + \lambda_1 \lambda_2 \left( \alpha A \gamma_k \right)
\]
with $A_3 = \alpha C_1 + (1 - \alpha) / (2Z)$, $A_4 = \alpha C_1' + (1 - \alpha) / (4Z)$, $A_5 = \alpha C_2 + (1 - \alpha) / (2Z)$, and the spin correlation functions $C_1 = (1/Z^2) \sum_{\vec{q}} \eta \gamma_1 \gamma_{\vec{q}}$, $C_1' = (1/Z^2) \sum_{\vec{q}} \eta \gamma_{\vec{q}} \gamma_{\vec{q} - \vec{r}}$, $C_2 = (1/Z^2) \sum_{\vec{q}} \gamma_{\vec{q} - \vec{r}} \gamma_{\vec{q} + \vec{r}}$, $C_3 = (1/Z) \sum_{\vec{q}} \gamma_{\vec{q} - \vec{r}} \gamma_{\vec{q} + \vec{r}}$, and $C_3' = (1/Z) \sum_{\vec{q}} \gamma_{\vec{q} + \vec{r}} \gamma_{\vec{q} - \vec{r}}$.

In order to satisfy the sum rule of the correlation function $\langle S_{1+\vec{r}}^z S_{-1+\vec{r}}^z \rangle = 1 / 2$ in the case without AFLRO, the important decoupling parameter $\tau$ has been introduced in the MF calculation [16,24], which can be regarded as the vertex correction [25].

The dressed spin Green’s function $\Sigma^{(d)}(k, \omega)$ in the SC-state with the $d$-wave symmetry is obtained from the dressed holon bubble in the dressed holon particle-particle channel as [17],
\[
\Sigma^{(s)}(k, i\omega_n) = \frac{1}{N^2} \sum_{p,q} A(q, p, k) \sum_{i\omega_m} D^{(0)(q + k, i\omega_m + i\omega_n)} \sum_{i\omega_m} \Sigma^{(d)}(p, i\omega_m) \Delta(p, q, i\omega_m + i\omega_n),
\]
with $A(q, p, k) = (Z\gamma_{-p} - Zt' \gamma_{-p+q})^2 + (Z\gamma_{q+p} - Zt' \gamma_{q+p})^2$, $N$ is the number of sites, and the dressed holon off-diagonal Green’s function [17,26],
\[
\Delta^{(d)}(k, i\omega_n) = -Z_F \Delta_{H}(k) \frac{1}{2E_{\xi}(k)} \left( \frac{1}{i\omega_n - E_{\xi}} - \frac{1}{i\omega_n + E_{\xi}} \right),
\]
where $\Delta_{H}(k)$ is $Z_F \Delta_{H}(k)$, the dressed holon quasiparticle spectrum $E_{\xi}(k) = \sqrt{\xi_{\xi}^2 + |\Delta_{H}(k)|^2}$, $\xi_{\xi} = Z_F \xi_{\xi}$, the MF dressed holon excitation spectrum $\xi_{\xi} = Zt\tau_k \gamma_{-p} - Zt' \chi_{-k} \gamma_{-p+q} - \mu$, $\Delta_{H}(k)$ is $\Delta_{H}(k)^{(d)}$ the effective dressed holon gap function in the $d$-wave symmetry, $\gamma_{-p}^{(d)} = (\cos k_x - \cos k_x) / 2$, and the quasiparticle coherent weight $Z_F$ and effective dressed holon gap parameter $\Delta_h$ are determined by the following two self-consistent equations [17],
\[
1 = \frac{1}{N^2} \sum_{q+p} (Z\gamma_{q+p} - Zt' \gamma_{q+p}) \frac{Z_F^2 B_k B_p}{E_k \omega_p \omega_q} \left( \frac{F_1^{(1)}(k, q, p)}{(\omega_p - \omega_q)^2} \right) - \frac{F_2^{(2)}(k, q, p)}{(\omega_p - \omega_q)^2 + E_p^2} + \frac{F_3^{(3)}(q, p)}{(\omega_p - \omega_q)^2} + \frac{F_4^{(4)}(q, p)}{E_{p-q}^2},
\]
\[
Z_F^2 = 1 + \frac{1}{N^2} \sum_{q+p} (Z\gamma_{q+p} - Zt' \gamma_{q+p}) \frac{Z_F B_k B_p}{4\omega_q \omega_p} \left( \frac{F_1^{(1)}(q, p)}{(\omega_p - \omega_q)^2} + \frac{F_2^{(2)}(q, p)}{(\omega_p - \omega_q)^2 + E_{p-q}^2} \right),
\]
where
\[
F_1^{(1)}(k, q, p) = \omega_p - \omega_q \left[ n_B(\omega_q) - n_B(\omega_p) \right] \left[ 1 - 2n_F(E_k) \right] + \left[ n_B(\omega_l) n_B(\omega_q - \omega_p) + n_B(\omega_q) n_B(\omega_p) \right] - n_B(\omega_p) n_B(\omega_q) n_B(\omega_p) n_B(\omega_q),
F_2^{(2)}(q, p) = n_F(E_{p-q}) \left[ n_B(\omega_q) - n_B(\omega_p) \right] - n_B(\omega_p) n_B(\omega_q),
F_2^{(3)}(q, p) = n_F(E_{p-q}) \left[ n_B(\omega_q) - n_B(\omega_p) \right] - n_B(\omega_p) n_B(\omega_q),
F_2^{(4)}(q, p) = n_F(E_{p-q}) \left[ n_B(\omega_q) - n_B(\omega_p) \right] - n_B(\omega_p) n_B(\omega_q).
\[ F_{2}^{(3)}(q, p) = n_{F}(E_{p+q})\left[ n_{B}(\omega_{q}) - n_{B}(-\omega_{q}) \right] + n_{B}(\omega_{p})n_{B}(\omega_{q}), \]
\[ F_{2}^{(4)}(q, p) = n_{F}(E_{p-q})\left[ n_{B}(-\omega_{q}) - n_{B}(\omega_{p}) \right] + n_{B}(-\omega_{p})n_{B}(-\omega_{q}), \]

\( n_{B}(\omega) \) and \( n_{F}(\omega) \) are the boson and fermion distribution functions, respectively, and \( k_{0} \approx k_{A} - k_{F} \) with \( k_{A} = [\pi, \pi] \) and \( k_{F} \approx [(1 - x)\pi/2, (1 + x)\pi/2] \) that guarantees \( Z_{F} \) near the electron Fermi surface \([17]\). Now the dressed spin self-energy function (5) can be evaluated explicitly in terms of the dressed holon off-diagonal Green’s function (6) and dressed spin MF Green’s function (3) as,

\[ \Sigma^{(s)}(k, \omega) = \frac{1}{N^{2}} \sum_{p, q} A(q, p, k) B_{q+k}(\omega_{q+k}) Z_{q}^{2} \bar{\Delta}_{h}(p) \bar{\Delta}_{h}(q + p) \left( \frac{F_{s}^{(1)}(k, p, q)}{\omega^{2} - (E_{p} - E_{p+q} + \omega_{q+k})^{2}} \right) + \frac{F_{s}^{(2)}(k, p, q)}{\omega^{2} - (E_{p+q} - E_{p} + \omega_{q+k})^{2}} + \frac{F_{s}^{(3)}(k, p, q)}{\omega^{2} - (E_{p} + E_{p+q} + \omega_{q+k})^{2}} + \frac{F_{s}^{(4)}(k, p, q)}{\omega^{2} - (E_{p+q} + E_{p} - \omega_{q+k})^{2}}, \] (8)

where

\[ F_{s}^{(1)}(k, p, q) = (E_{p} - E_{p+q} + \omega_{q+k})\left[ n_{B}(\omega_{q+k})\left[ n_{F}(E_{p}) - n_{F}(E_{p+q}) \right] - n_{F}(E_{p+q})n_{F}(-E_{p}) \right], \]
\[ F_{s}^{(2)}(k, p, q) = (E_{p+q} - E_{p} + \omega_{q+k})\left[ n_{B}(\omega_{q+k})\left[ n_{F}(E_{p}) - n_{F}(E_{p+q}) \right] - n_{F}(E_{p+q})n_{F}(-E_{p}) \right], \]
\[ F_{s}^{(3)}(k, p, q) = (E_{p} + E_{p+q} + \omega_{q+k})\left[ n_{B}(\omega_{q+k})\left[ n_{F}(-E_{p}) - n_{F}(E_{p+q}) \right] + n_{F}(E_{p+q})n_{F}(-E_{p}) \right], \]
\[ F_{s}^{(4)}(k, p, q) = (E_{p} + E_{p+q} - \omega_{q+k})\left[ n_{B}(\omega_{q+k})\left[ n_{F}(-E_{p}) - n_{F}(E_{p+q}) \right] + n_{F}(E_{p+q})n_{F}(-E_{p}) \right]. \]

With the help of the full dressed spin Green’s function (2), we can obtain the dynamical spin structure factor of the \( t-t'-J \) model in the SC-state with the \( d \)-wave symmetry as,

\[ S(k, \omega) = -2\left[ 1 + n_{B}(\omega) \right] \text{Im} D(k, \omega) = -\frac{2\left[ 1 + n_{B}(\omega) \right] B_{k}^{2} \text{Im} \Sigma^{(s)}(k, \omega)}{\omega^{2} - \omega_{k}^{2} - B_{k} \text{Re} \Sigma^{(s)}(k, \omega)} + [B_{k} \text{Im} \Sigma^{(s)}(k, \omega)], \] (9)

where \( \text{Im} \Sigma^{(s)}(k, \omega) \) and \( \text{Re} \Sigma^{(s)}(k, \omega) \) are the imaginary and real parts of the second-order dressed spin self-energy function (8), respectively.

Within the \( t-t'-J \) model, we [26] have shown that although the symmetry of the SC-state is doping dependent, the SC-state has the \( d \)-wave symmetry in a wide range of doping, around the optimal doping \( x_{\text{opt}} = 0.15 \). In this case, we plot the dynamical spin structure factor \( S(k, \omega) \) of the \( t-t'-J \) model in the \((k_{x}, k_{y})\) plane in the optimal doping \( x_{\text{opt}} = 0.15 \) with temperature \( T = 0.002J \) for parameters \( t/J = 2.5 \) and \( t'/t = 0.3 \) at energy (a) \( \omega = 0.12J \), (b) \( \omega = 0.4J \), and (c) \( \omega = 0.82J \) in Fig. 1. As in the \( t-J \) model [17], the distinct feature of the present result in Fig. 1 is the presence of the IC-commensurate-IC transition in the spin fluctuation geometry, where the magnetic excitations disperse with energy, i.e., although at low energy the magnetic scattering peaks are found at \(([(1 + \delta)/2, 1/2] \) and \([1/2, (1 + \delta)/2] \) (hereafter we use the units of \([2\pi, 2\pi] \)), the positions of these parallel IC magnetic scattering peaks move towards \([1/2, 1/2] \) with increasing energy, and then the commensurate \([1/2, 1/2] \) magnetic resonance peak appears at intermediate energy \( \omega_{0} = 0.4J \). Using an reasonably estimative value of \( J \approx 100 \text{ meV} \) in cuprate superconductors [27], the present result of the magnetic resonance energy \( \omega_{0} = 0.4J \approx 40 \text{ meV} \) is in quantitative agreement with the resonance energy \( \approx 41 \text{ meV} \) observed in the optimally doped \( \text{YBa}_{2}\text{Cu}_{3}\text{O}_{6+y} \) [79–11]. In comparison with the result of the magnetic resonance energy \( \omega_{r} = 0.35J \approx 35 \text{ meV} \) obtained from the \( t-J \) model [17], our present result of the \( t-t'-J \) model also shows that the additional second neighbor hopping \( t' \) is systematically accompanied with the enhancement of the magnetic resonance energy, and therefore plays an important role in quantitatively determining the magnetic resonance energy. For a better understanding of the commensurate \([1/2, 1/2] \) magnetic resonance, we have discussed the doping dependence of the commensurate \([1/2, 1/2] \) magnetic resonance, and the result of the magnetic resonance energy \( \omega_{r} \) as a function of doping \( x - x_{\text{opt}} \) in Fig. 2 in comparison with the experimental result [9] (inset). It is shown that in analogy to the doping dependence of the SC transition temperature [26], the magnetic resonance energy \( \omega_{r} \) increases with increasing doping in the underdoped regime, and reaches a maximum in the optimal doping, then decreases in the overdoped regime. Furthermore, this commensurate \([1/2, 1/2] \) magnetic resonance peak is separated again at high energy (above the magnetic resonance energy), where all IC magnetic scattering peaks lie on a circle of radius of \( \delta' \), with the value of \( \delta' \) is different from the corresponding value of \( \delta \) at low energy. As in the \( t-J \) model [17], although some IC satellite parallel peaks appear, the main weight of the IC magnetic scattering peaks is in the diagonal direction. Moreover, the separation at high energy gradually increases with increasing energy although the peaks have a weaker intensity than those below the magnetic resonance energy. To show the unusual double dispersion of the magnetic excitations clearly, we plot the evolution of the magnetic scattering peaks with energy at \( x_{\text{opt}} = 0.15 \) in Fig. 3. For comparison, the experimental result [11] of \( \text{YBa}_{2}\text{Cu}_{3}\text{O}_{6+y} \) with \( y = 0.7 \) (\( x \approx 0.12 \)) in the SC-state is also shown in the same figure. The similar experimental results [10,12] have also been obtained for \( \text{YBa}_{2}\text{Cu}_{3}\text{O}_{6+y} \) with different doping concentrations. In comparison with the result of the \( t-t'-J \) model [17], our present result shows that although there is still a narrow energy range for the commensurate \([1/2, 1/2] \) magnetic resonance peak, this narrow energy range for the commensurate \([1/2, 1/2] \) magnetic resonance...
Fig. 1. The dynamical spin structure factor $S(k, \omega)$ in the $(k_x, k_y)$ plane at $x_{\text{opt}} = 0.15$ with $T = 0.002J$ for $t/J = 2.5$ and $t'/t = 0.3$ at (a) $\omega = 0.12J$, (b) $\omega = 0.4J$, and (c) $\omega = 0.82J$.

Fig. 2. The magnetic resonance energy $\omega_r$ as a function of $x - x_{\text{opt}}$ with $T = 0.002J$ for $t/J = 2.5$ and $t'/t = 0.3$. Inset: the experimental result taken from Ref. [9].

peak has been reduced in the present $t-t'-J$ model. On the other hand, this similar narrow energy range for the commensurate magnetic resonance peak has been observed from experiments [10]. Our result also shows that in contrast to the case at low energy, the magnetic excitations at high energy disperse almost linearly with energy [11–13]. In the framework of the kinetic energy driven SC mechanism [16], these mediating dressed spin excitations in the SC-state are coupled to the conducting dressed holons (then electrons), and have energy greater than the dressed holon pairing energy (then Cooper pairing energy), and are present at the
our these results of the \( \omega \) SC transition temperature \[17\]. Our these results of the \( W(k,\omega) \) observations of doped cuprates in the SC-state \[7,9–13\].

On the other hand, the dynamical spin structure factor \( S(k,\omega) \) character, where \( \Sigma(k,\omega) \) is defined renormalized spin excitations. Since the essential physics is dominated by the dressed spin self-energy renormalization is a strong angular dependence with actual minima in \( [1 \pm \delta/2,1/2] \) for low, intermediate, and high energies, respectively. These are exactly positions of the IC magnetic scattering peaks at both low and high energies and commensurate \( [1/2,1/2] \) magnetic resonance peak at intermediate energy, i.e., the mechanism of the IC magnetic scattering and commensurate \( [1/2,1/2] \) magnetic resonance in the SC-state is most likely related to the motion of the dressed holon pairs (then the electron Cooper pairs). On the other hand, \( t' \) describes hopping within the same magnetic sublattice in the AF background,
and it does not alter the AF properties of the systems, while at the same time, it is not strongly renormalization by AFSRC but contributes directly to the coherent motion of the dressed holon pair in the SC-state \[5\], and therefore has a substantial impact on the spin response in the SC-state. This is why the positions of the IC magnetic scattering peaks and commensurate \[1/2, 1/2\] magnetic resonance peak in the SC-state can be quantitatively determined in the present study within the \(t-t'-J\) model based on the kinetic energy driven SC mechanism, while the dressed spin energy dependence is ascribed purely to the self-energy effects which arise from the dressed holon bubble in the dressed holon particle–particle channel.

In summary, we have discussed the effect of the additional second neighbor hopping \(t'\) on the doping and energy dependent spin response of doped cuprates in the SC-state under the kinetic energy driven SC mechanism. Within the \(t-t'-J\) model, we have performed a systematic calculation for the dynamical spin structure factor of doped cuprates in the SC-state with the \(d\)-wave symmetry in terms of the collective mode in the dressed holon particle–particle channel, and quantitatively reproduced all main features found in the inelastic neutron scattering experiments on cuprate superconductors, including the energy dependence of the IC magnetic scattering at both low and high energies \[11–13\] and commensurate \[1/2, 1/2\] magnetic resonance at intermediate energy \[9,10\]. In particular, the unusual IC magnetic excitations at high energy disperse almost linearly with energy, and have energies greater than the dressed holon pairing energy (then SC Cooper pairing energy), and are present at the SC transition temperature. Our result also shows that the additional second neighbor hopping \(t'\) is systematically accompanied with the enhancement of the magnetic resonance energy, and therefore it plays an important role in quantitatively determining the magnetic resonance energy.

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