Anomalous quantum glass of bosons in a random potential in two dimensions

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References:
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Introduction

QMC simulations of site-disordered BHM in 2D

Percolation scenario and avoided degeneracies

Critical behavior at SF-QG transition

Summary
Interacting lattice bosons: clean system

The Bose-Hubbard model

\[ H = -t \sum_{\langle ij \rangle} (b_i^+ b_j + b_j^+ b_i) + \frac{U}{2} \sum_{i=1}^{N} n_i(n_i - 1) - \mu \sum_{i=1}^{N} n_i, \]

two ground states: Superfluid and Mott insulator

\[ |\psi_{SF}\rangle \propto (\sum_i^{M} a_i^+)^N |0\rangle, \quad \langle a_i \rangle \neq 0 \]
delocalized, macroscopic phase

\[ |\psi_{MI}\rangle \propto \prod_i^{M} (a_i^+)^n |0\rangle, \quad \langle a_i \rangle = 0 \]
localized, no macroscopic phase
Interacting lattice bosons: clean system

Phases vary as functions of $\mu/U$ and $t/U$

- **Mott gap**: the width of the platform

**Phase diagram**

- **MI**: $\kappa = 0, \rho_s = 0$; **SF**: $\kappa \neq 0, \rho_s \neq 0$

**Physical observables**:
- **Compressibility**:
  \[ \kappa = \frac{\partial \rho}{\partial \mu} = \frac{1}{NT} \left[ \langle n^2 \rangle - \langle n \rangle^2 \right] \]
- **Superfluid stiffness**:
  \[ \rho_s \propto \langle W^2 \rangle \], determined by response to boundary motion

**Two ways from MI to SF**

\[ \langle a_i^+ a_j \rangle \propto \exp(-|i-j|/\xi) \]
Interacting lattice bosons: disorder present

Fascinating interplay between disorder, interactions and SF

- two kinds of disorder: hopping disorder and site disorder
- we focus on site disorder: no explicit particle-hole symmetry

\[ H = -t \sum_{\langle ij \rangle} (b_i^+ b_j + b_j^+ b_i) + \frac{U}{2} \sum_{i=1}^{N} n_i(n_i - 1) - \sum_{i=1}^{N} \mu_i n_i, \]

Bounded disorder

- Quantum Glass: localized but Gapless
Previous studies of the site-disordered BH model

$T = 0$ phase diagram:

In the presence of disorder, QG always intervenes SF and MI?

(a) Söyler '11; Prokof'ev, '04; Pollet, '09; Herbut, '97; '98; Weichman, '96, '08; Svistunov, '96;

(b) Singh, '92; Pazmandi, '98; Pai, '96;

(c) Scalettar, '91; Krauth, '91; Kisker, '97; Sen, '01; Lee, '01; Wu, '08; Bissbort, '09
Recent progresses: theorem of inclusion

Recent progresses: theorem of inclusion

Clean system

With disorder

QG always between SF and MI

Most studies focused on $\rho = 1$, in the $U - \Lambda$ plane

Pollet et al, PRL, 2009; Söyler et al, 2011; Rieger, meanfield phase diagram, NJP, 2013
\( \rho = 1 \) 2D BHM with site disorder

The phase diagram is well established, the properties of QG state for 2D BHM with site disorder are not well understood

\[ \kappa = \frac{\partial \rho}{\partial \mu} = \frac{1}{NT} \left[ \langle n^2 \rangle - \langle n \rangle^2 \right] \]

two types of glass states are known;
- the compressible Bose glass (BG)
- the incompressible Mott glass (MG)

Commonly believed:
- MG only at commensurate filling with particle-hole symmetry
- BG in the 2D site-disordered BHM, always compressible
QG is a Griffiths phase: **rare large regions** of phase A (SF) inside phase B (MI) lead to singularities.

**Griffiths arguments:** 
- MI-QG boundary based on $\Lambda = \Delta_M / 2$
- $\Lambda > \Delta_M / 2$, arbitrarily large SF lakes can appear
- $\Lambda < \Lambda_c$, SF lakes NOT percolating, localized

*Fisher 89, Freericks 96*
Standard Scenario for compressible BG

- there are steps in the $\langle n_i \rangle$ vs $\mu_i$ curve → gapless points
- contribution from lake $\mu_i$ in a window $T$.
- within this window $\langle n_i^2 \rangle - \langle n_i \rangle^2 \approx 1$
- the probability of being inside the window is $T/\Lambda$, on average a lake contributes: $1/(N\Lambda)$
- the number of lakes $\propto N$, thus $\kappa$ finite while $T \to 0$.

- Fundamentally different from the MI: gapless and nonzero compressibility BG

Pollet et al, PRL, 2009; Gurare et al PRB, 2009
\( \rho = 1 \) 2D BHM with site disorder, QMC study

\( L = 16, \beta = 8 \)

Using SSE QMC, we study

- compressibility \( \kappa \): particle-number fluctuations
- superfluid stiffness \( \rho_s \): winding number fluctuations

Parameters:

- Fix (a) \( U/t = 22 \) (b) \( U/t = 60 \)
- Adjust \( \mu \) to ensure \( \rho = \langle n \rangle / N = 1 \)
- Average over up to hundreds of realizations
Our MC results

- along $U/t = 22$

- $\rho_s$:
  - sharp increase at $\Lambda \approx 8$, enter SF
  - decreases to zero at $\Lambda \approx 30$, enter QG.

- along $U = 60$

  - $\kappa$ increases rapidly with $\Lambda$ between 28 to 31 (MI: $\Lambda < 24$)

- $\kappa$ changes more than four orders of magnitude between BG and MG

$L = 16, \beta = 8$

- $\rho_s$, $\kappa$
  - substantial in SF and QG re-entered at large $\Lambda$.
  - However, it is very small before entering the SF phase, not only in the Mott phase but also in QG (QG for $4.2 < \Lambda < 7.8$)
QMC simulations: $\kappa - T$ behavior

- Quantum Glass region

$$\kappa \sim \exp(-b/T^\alpha) + c, \quad \alpha < 1, \, c = 0$$

- comparing to MI: the above form with $\alpha = 1$ and $b = \Delta$

$L = 32, \, U/t = 22$

$\kappa$ in QG (MG) follows the exponential form

$$\kappa \sim \exp(-b/T^\alpha)$$

$\alpha \approx 0.77$ for $\Lambda = 6$

$\alpha \approx 0.53$ for $\Lambda = 7$

- MI points ($\Lambda = 0$ and $3$):

  $\kappa \propto e^{-\Delta/T}$

- SF point ($\Lambda = 9$):

  $\kappa(T)$ converges rapidly to a non-zero value
**BG: \( \kappa - T \) behavior**

- compressible BG in the re-entrance region

\[ U = 22, \Lambda = 60 \]
MG: temperature behavior of $\kappa$

- $\chi$ in random quantum spin systems, corresponding to $\kappa$, vanishes as $T \to 0$ due to spin-inversion symmetry, corresponding to particle-hole symmetry for bosons

  Roscilde, PRL 2007, Ma 2014

- Such an incompressible and insulating QG is termed an MG and has also been shown to exist in variants of the 2D random BHM where particle-hole symmetry is explicitly built in


- In the presence of random potentials there is no explicit particle-hole symmetry. But, in principle there could be emergent particle-hole symmetry, as in the clean BHM at the tips of the Mott lobes

- When $T \to 0$, $\kappa = 0$ may not hold strictly, the physics behind is similar to a true Mott Glass.
Percolation scenario: explain the temperature behavior

Consider an ensemble of SF lakes below the percolation threshold

- Finite-size $(s)$ gap $\propto \frac{1}{s^a}$, $a$ unknown exponent.
- Given $T = \frac{1}{m^a}$, all lakes of sizes $s < m$ gapped, NO contribution to $\kappa$
- Only lakes of sizes $s > m$ contribute to $\kappa$

The prob. of a site belonging to a lake of size $s > m$ is $\propto \exp(-bm^c)$, $c$ unknown exponent.

Thus

$$\kappa \propto \exp(-bT^{-c/a}) = \exp(-bT^{-\alpha})$$
How about the standard scenario?

- The percolation scenario neglects the arbitrarily close degeneracy of different particle-number sectors

- How can these degeneracies be avoided?
Avoided degeneracy

study isolated lakes embedded in Mott background

- finite-size effects:
  - particle-number degeneracy occurs at much larger $\mu_i$ than in thermodynamic limit.
  - or, given a $\Delta \mu_A > \Delta_M/2$, the lake has to be larger than a critical size $L_A^c$ to reach the degeneracy.
  - $L_A^c$ diverges at the Mott phase boundary

- all lakes below $L_A^c$ do not contribute to $\kappa$ and should not be regarded as superfluid
Avoided degeneracy

- Effect of cluster shape

for a given deviation $\Delta \mu_A > \Delta M/2$, $L_A^C$ of an irregularly shaped cluster is even larger than that of the squared one
rare lakes exceeding the critical size lead to small compressibility?

- Within the standard scenario, there should still exist rare large compressible lakes in the Mott background. but the lakes are
  - never completely isolated from the background

\[
\langle (n_i + n_B)^2 \rangle - \langle n_i + n_B \rangle^2 \neq \langle n_i^2 \rangle - \langle n_i \rangle^2
\]

- never completely isolated from each other.

\[
\left[ \langle \sum_{i=1}^{N_l} n_i \rangle^2 \right] - \left( \langle \sum_{i=1}^{N_l} n_i \rangle \right)^2 \neq \sum_{i=1}^{N_l} \left[ \langle n_i^2 \rangle - \langle n_i \rangle^2 \right]
\]

spectral structure altered due to quantum-criticality when the SF boundary is approached.
Critical behavior at SF-MG transition ($T = 0$)

- Scaling at the quantum transition point
  \[ \kappa(\Lambda) \propto (\Lambda - \Lambda_c)^{\nu(2-z)} \]
  if $z = 2$ as often argued (Söyler, PRL, 2013),
  then $\kappa \neq 0$ at $\Lambda_c$ and inside glass
  if $z < 2$ then $\kappa = 0$ at $\Lambda_c$.

- A key question then is whether $z = 2$ or $z < 2$
  divergent SF clusters in the MI background close to the
  percolation point would be
  - $z = 2$ compressible,
  - $z < 2$ incompressible.

- There are arguments for $z = 2$ but no rigorous proofs.
- Some works on models related to BHM have $z < 2$
  Meier, PRL 2012; Priyadarshee, PRL 2006;
  some suggest $z = 2$, but with large error, also consistent
Critical behavior at SF-MG transition ($T = 0$)

- Determine the dynamic exponent $z$ according to FSS
  \[ \kappa_u(\Lambda, L) \propto L^{z-d} \kappa_u(\Lambda - \Lambda_c), \quad \rho_s(\Lambda, L) \propto L^{-z} \rho_s(\Lambda - \Lambda_c), \]
  $\kappa_u(\Lambda, L)$ and $\rho_s(\Lambda, L)$ are calculated at $\beta \approx L^{z}$.
- Comparing three conjectured $z = 2$, $1.75$, and $1.5$

- At $T = 0$, $\kappa \sim (\Lambda - \Lambda_c)^{\nu(2-z)}$, $z \approx 1.75$, continuous
- Consistent with the drop of $\kappa$ at SF-MG from right, while at SF-BG there are no strong variations, suggesting $z = 2$
Summary

- Based on plausible arguments and unbiased QMC results: there is a novel QG state with extremely small $\kappa$ decaying with temperature for commensurate filling and moderate disorder strength in 2D.

- Percolation scenario: finite-size gap of SF domains explains the decay of $\kappa$ with temperature

- Finite-size effects also make particle-number degeneracies avoided

- A dynamic exponent $z < 2$ provides an explanation for an anomalously small, or possibly vanishing, $T = 0$ compressibility in the finger region

- The sharp cross-over from anomalously small to normal compressibility away from the SF phase at larger $U$ also shows that there are two distinct types of glass phases

- The scenario in this work applies only to integer filling fractions

Thank You