Relevance of Deconfined-Criticality Action in the Light of the J-Q Spin Model

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Neel phase and VBS phase for 1/2-spin model

Neel order parameter: Neel Vector
break spin rotational symmetry

VBS order parameter
break translational lattice symmetry
Deconfined quantum criticality

Landau-Ginzburg paradigm:
- actions in terms of order parameter fields
- first order transition

Deconfined Critical Point (DCP) theory:
- deconfinement of spinons, Neel vector: $\vec{n} = z_\alpha \vec{\sigma}_\alpha \beta z_\beta$
- emergent non-compact U(1) gauge field
- $L_z = \sum_{\alpha=1}^{2} |(\partial_\mu - ia_\mu)z_\alpha|^2 + s|z|^2 + u(|z|^2)^2 + \kappa(\epsilon_{\mu\nu\kappa}\partial_\nu a_\kappa)^2$
- continuous phase transition

T. Senthil, et. al.
Heisenberg model with 4-spin interaction

SU(2)-symmetric J-Q model

\[ H = J \sum_{<ij>} S_i \cdot S_j - Q \sum_{<ijkl>} (S_i \cdot S_j - \frac{1}{4})(S_k \cdot S_l - \frac{1}{4}) \]

- has an AF-VBS phase transition at \( J/Q \approx 0.04 \)
- has emergent U(1) symmetry at \( J/Q = 0.0 \)

PRB 85, 134407 (2012), A. W. Sandvik
PRL 98, 227202 (2007), A. W. Sandvik
Q: Can the DCP theory describe the phase transition in the J-Q model?

A: Yes!

The DCP theory captures the mesoscopic physics of the phase transition in the J-Q model.
Comparison of 3D NCCP\(^1\) model and J-Q model

- 3D classical SU(2)-symmetric discrete NCCP\(^1\) model on a simple cubic lattice (DCP model)

\[
H_{DCP} = -t \sum_{\langle ij \rangle, a} (\psi_{ai}^* \psi_{aj} e^{iA_{\langle ij \rangle}} + c.c.) + \frac{1}{8g} \sum_P (\nabla \times A)^2
\]

- high-T expansion \(\Rightarrow\) J-current model
- worm algorithm

- 2D quantum SU(2)-symmetric J-Q model on square lattice

- path integral representation with continuous imaginary time
- worm algorithm

Configuration space and winding number

\[ R(L) = \langle \vec{W}^2 \rangle \]
Examples for first and second order flows

flows of $R(L) = \langle \vec{W}^2 \rangle$ at the critical point
Flowgram method


- finite size scaling function
  \[ R(L) = \langle W^2 \rangle = \Phi(L/\xi) \]

  - universal/quasi-universal for continuous/weak first order transition

- perform rescaling on flows of \[ R(L) \Rightarrow R(L/\xi) \] for different model

- “visualize” universality/quasi-universality by rescaling
Flowgrams of the J-Q and DCP models

DCP: \[ R(L, g) = \langle W_x^2 + W_y^2 + W_z^2 \rangle \]

J-Q: \[ R_{JQ}(L) = \langle W_x'^2 + W_y'^2 + W_\tau'^2 \rangle \]
Rescaled flowgrams

\[ DCP : \frac{L}{\xi(g)}; \quad JQ : \frac{L}{\xi_{JQ}} \]

The flows of two models:

- collapse in a broad region (for J-Q model, 10 < L < 50)
- depart at larger sizes
- both violate the scale invariance

The DCP theory captures the mesoscopic physics of competition between Neel and VBS orders!
But, at large system size \((L>50)\), the two flows depart...

The flow of DCP model ends up in first order phase transition:

Possible scenarios for J-Q flow:
- second order transition unlikely
- weak first order transition likely

Summary

✶ The collapse at intermediate region:

The DCP theory captures the essence of the phase transition in J-Q model at intermediate scales.

✶ The significant deviation at large scales:

At least one of the two models does not feature the second-order criticality.

Most likely the two models both feature weak first-order transition.

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