量子相变与解禁闭量子相变介绍

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提纲

相变基础知识

相变的统计物理描述

临界现象模型与理论
- Ising model, 关联函数
- 数值模拟: 蒙特卡洛方法
- 序参量与关联函数
- LGW 连续场论描述
- 标度理论与重整化群

量子临界现象
- 量子相变与模型
- 经典量子映射
- 场论描述与有限尺寸标度

退禁闭量子临界性介绍
- 拓扑相变和拓扑激发
- 理论介绍
- 微观模型及其数值研究

- Spin-liquid-like state in disordered 2D quantum magnets

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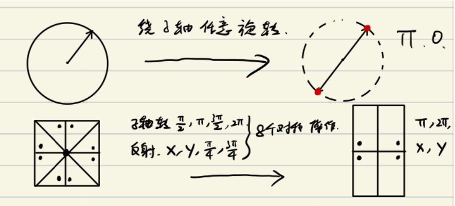
- 相（Phase）：

热力学系统（或它的一部分）如果具有均匀的物理性质，称为处于某相

- 传统上物态分类: 固相，液相，气相, 等离子体
- 现代观点(Landau): 相由对称性的破缺刻画



固体：晶格破缺了空间平移和旋转对称性，不同于液体
不同的晶格结构视为不同的相：破缺对称性的方式不同
液体和气体都没有破缺平移和旋转对称性，实际是一个相

- 相变(Phase Transition)：物质性质的不连续或者不光滑，突兀的转变
  - 从一种相转变成另一种相: 比如冰融化就是水的固液相变
  - 同一个相也可以：气液相变（100度以下烧水，可避免)

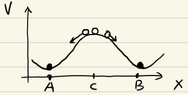
丰富多彩的相变与临界现象

- 热力学相变（或经典相变）：温度或者说热带来涨落，热涨落驱动
  - 气液固三态之间的转变
  - 铁磁, 顺磁, 反铁磁之间的转变
  - 超导体与正常导体的转变
  - 超流与正常流体的转变
- 量子相变：零温, 没有热涨落, 由量子涨落驱动的转变
  由调节外参量，改变基态. 外参量可以是：压力, 磁场强度, 参杂浓度。
  - BEC的超流-Mott绝缘体转变
  - 铁磁到量子顺磁
  - Neel 反铁磁到量子顺磁
  - 磁性的绝缘体通过掺杂转变成高温超导体

相变与临界现象总伴随着热力学量的**奇异性**:

不连续、发散或导数发散！

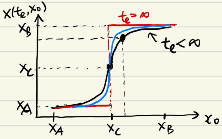
熟悉的物理: $\frac{dx}{dt}, \frac{d^2x}{dt^2}, \nabla V$ 可导，光滑



小球从C附近滚落，过阻尼 $m=1,\ r=1$

$$\frac{dx}{dt} = -\frac{\partial V(x)}{\partial x}$$

初始 x_0，末时刻 t_e 的位置 $x(t_e,x_0) = -\int_0^{t_e} \frac{\partial V(x)}{\partial x}\, dt + x_0$



- $x(t_e,x_0)$ 是 x_0 的连续函数，若 t_e 有限 (finite)
- 当 $t_e \to \infty$，变成不连续：x_0 偏离 x_C 无穷小，都导致末位置为 A 或 B：阶跃！

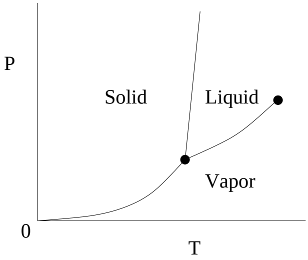
大量粒子的集体行为(类比于长时间行为) 可以非常不同于单个粒子(类比于有限时间行为)，

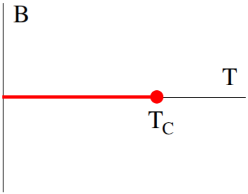
宏观性质变化可以不光滑: 相变

现代物理学把相变分成两类

- **一级相变(First Order Phase Transition)**：

自由能的一级导数不连续，两相（或多相）共存

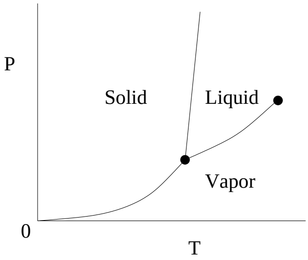


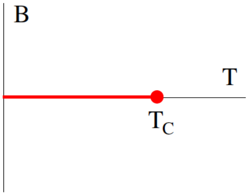


现代物理学把相变分成两类

- **连续相变(continuous phase transition),**

  对称性自发破缺， 系统性质(对称性)整体转变, 不是一个相从另一个相中‘生长’ 出来





- 连续相变点称为临界点(Critical Point): 高温对称相小磁铁方向乱, 低温对称破缺：指向特定方向
- 系统表现出非常特殊的性质称为 临界现象(Critical Phenomena)
  - 通常比热，磁化率等自由能二级导数发散, 又称二级相变

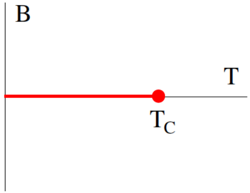
临界指数

临界现象更有兴趣！

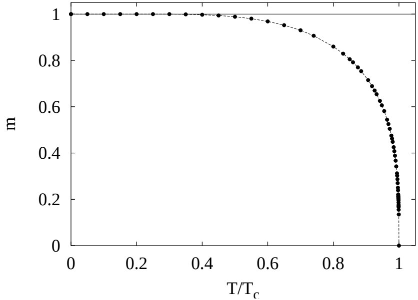
热力学量的发散可以用临界指数来描述

临界指数

以单轴各向异性铁磁体为例



自发磁化



- 磁化强度

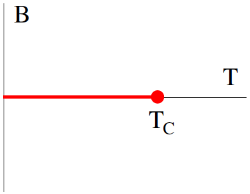
$$m \propto t^{\beta} \qquad t = (T_c - T)/T_c$$

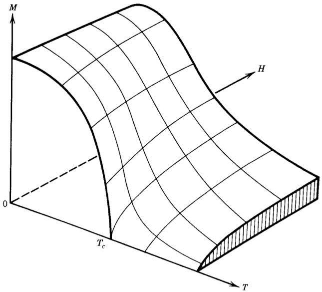
其中 T_c: 居里点，临界点

- 临界指数

$$\boxed{2D : \beta = 1/8; \qquad 3D : \beta = 1/3}$$

临界指数





from K.S. Huang

- 靠近T_c, 磁化率

$$\chi = \frac{\partial m}{\partial h} \propto t^{-\gamma}, \qquad t = (T_c - T)/T_c$$

  - 临界指数

$$\boxed{2D : \gamma = 7/4; \quad 3D : \gamma = 4/3}$$

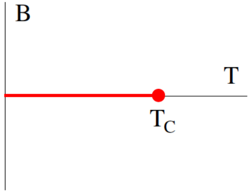
- 在T_c处,

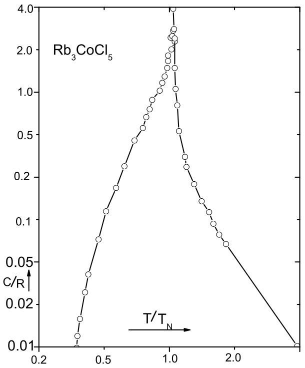
$$h \propto m^{\delta}$$

  - 临界指数

$$\boxed{2D : \delta = 15; \quad 3D : \delta = 5}$$

临界指数





比热

- 3D 情况

$$c \propto |t|^{-\alpha}, \qquad t = (T_c - T)/T_c$$

► 临界指数

$$\boxed{\alpha \approx 0.1096 \pm 0.0005}$$

自然界里的新常数?

- 2D 特殊, 对数发散

$$c \propto -\ln(t).$$

► 临界指数

可以认为$\alpha = 0$.

普适类

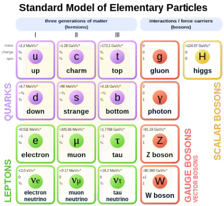
- 上面的现象在所有临界点都类似，但临界指数可以不同
- 很多完全不同的相变却有相同的临界指数－**普适类**
  汽液临界点α与单轴铁磁体(3D)的一样！
- 核心的问题是对称性, 与细节无关

  讨论相变问题发展出来的理论和想法都可以直接应用到粒子物理，宇宙学.....

  比如真空，可以处于不同的相, 大爆炸后冷却过程中发生多次相变.....

普适性的出现还与尺度有关：Nature is organised by Scale

- little things affect big things： 粒子→ 核→ 原子→ 化学或凝聚态系统→ · · ·
  而不会相反：星象学
- but rarely affect very big things, but slightly bigger things, and so on
  研究椋鸟群聚飞行行为的动物学家不需要研究基本粒子的规律
  牛顿，Einstein 不需要知道量子力学就可以写下他们的方程



- 通过研究相变，人们提出了重整化理论，通过它我们可以同时理解不同尺度(scale)的物理：Higgs boson 和椋鸟的群聚
  在不同的尺度上涌现（emerge)出该尺度上的first principle

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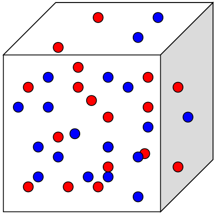
相互作用多粒子系统的描述: 统计物理

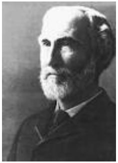


- ▶ **微观状态**

- 经典系统，可认为任意力学量与H对易, 每个微观状态Γ有确定能量$E(\Gamma) = H(\Gamma)$

- **平衡态配分函数(partition function)**

$$Z = \sum_{\Gamma} \mathrm{e}^{-E(\Gamma)/k_B T} = \sum_{\Gamma} W(\Gamma)$$



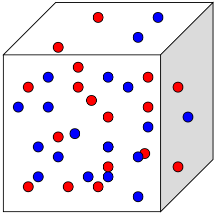
每个微观状态出现的几率由温度和该状态的能量决定:
正则分布(Canonical distribution)

$$p_{\mathrm{eq}}(\Gamma) = W(\Gamma)/Z$$

物理量的统计期望值: $\langle A \rangle = \sum_{\Gamma} A(\Gamma) \frac{W(\Gamma)}{Z}$

P_{eq}带来涨落(同样的系统，同样的参数，不一样的物理量)
本质是热涨落

相互作用多粒子系统的描述: 统计物理



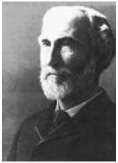
- **微观状态**
  - 量子系统，哈密顿量的本征态不容易知道

- **平衡态配分函数**

$$Z = \mathrm{Tr}\,\mathrm{e}^{-H/k_BT} = \sum_{\alpha}\langle\alpha|\mathrm{e}^{-\beta H}|\alpha\rangle$$

$|\alpha\rangle$是任意正交完备基矢(也可以是$|E\rangle$)
物理量的统计期望值

$$\langle A\rangle = \frac{1}{Z}\mathrm{Tr}\{A\mathrm{e}^{-\beta H}\}$$



当温度$T\to 0$, $\langle A\rangle = \langle 0|A|0\rangle$

当A与H没有共同本征态时，带来涨落，这是量子涨落(系统处于同样的量子态，可是测量结果可以不同)

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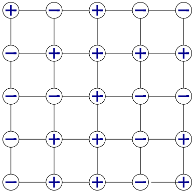
Ising model (经典), 描述相变的真空球形鸡

描写单轴铁磁体（可以在任意晶格上，以二维正方晶格为例）



$$H = -J\sum_{\langle i,j \rangle} s_i s_j - B\sum_k s_k; \quad s_k = \pm 1$$

微观状态$\Gamma = (s_1, s_2, \cdots, s_N)$.
总磁矩$M(\Gamma) = \sum_k s_k$

- $J > 0$, 铁磁(ferromagnetic)
- $J < 0$, 反铁磁(antiferromagnetic)

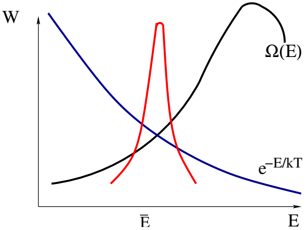
当系统处于热平衡

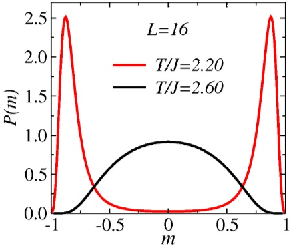
$$\langle M \rangle = \sum_{\Gamma} M(\Gamma) p_{eq}(\Gamma)$$

- 在磁性和相变理论中非常重要
- 也是其它统计物理问题的有效模型：格气(lattice gas), 合金，原子在表面的吸附问题等

如果没有外磁场,

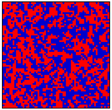
$$Z = \sum_{\Gamma} e^{-\beta H} = \sum_{E} \Omega(E) \mathrm{e}^{-E/k_B T}, \qquad \beta = 1/k_B T$$

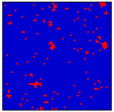




- 自由能密度

$$f = -\frac{k_B T}{N} \ln Z \approx \frac{\bar{E}}{N} - k_B T \frac{\ln \Omega(\bar{E})}{N}$$



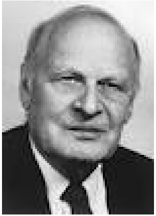


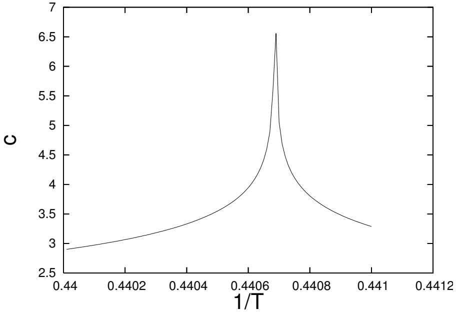
- 能量与熵的竞争: 高温无序, $\bar{E} \to \Omega$极大, 低温有序$\bar{E} \to E$极小

Onsager 的里程碑

看似简单的Ising模型，求解异常困难

对二维Ising模型的求解是统计物理理解相变问题的里程碑！

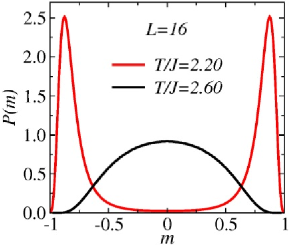


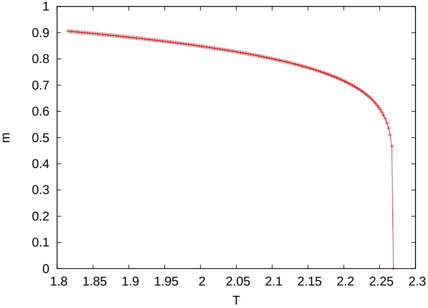


临界点附近比热对数发散

$$c \propto k_B \ln |(T - T_c)/T_c|$$

杨振宁的磁化强度





临界温度以下$T < T_c$:

$$m = (1 - \frac{1}{\sinh^4 2/T})^{1/8} \propto (\frac{|(T - T_c)|}{T_c})^{1/8}$$

指数严格为1/8

数值方法

Monte Carlo simulation: 直接求和或积分不可能或不容易时的办法

以Ising model 为例，考虑任意物理量A, 计算它的统计平均

$$\langle A \rangle = \sum_{\Gamma} A(\Gamma) p(\Gamma)$$

直接求和不现实：2^N个微观状态（位形）

重要性抽样(Importance sampling)

[Metropolis, Rusenbluth, Rosenbluth, Teller, and Teller, Phys.Rev.1953]

- 构造一个**随机过程**，得到一系列微观状态$\Gamma_1, \Gamma_2, \cdots, \Gamma_M$.

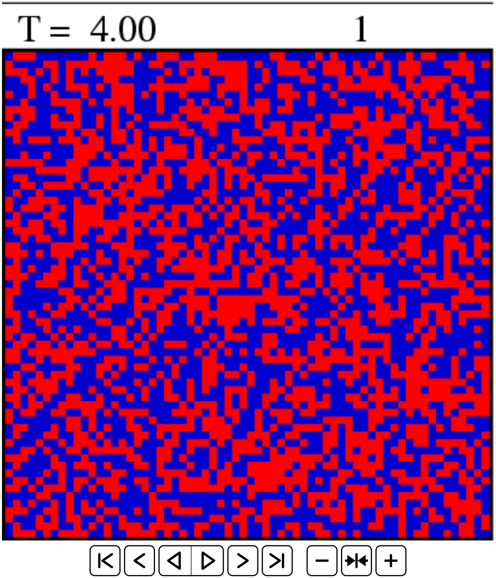
  Γ_{t+1}由Γ_t按照一定的跃迁几率(transition prob.)$T(\Gamma_{t+1}, \Gamma_t)$ 得到

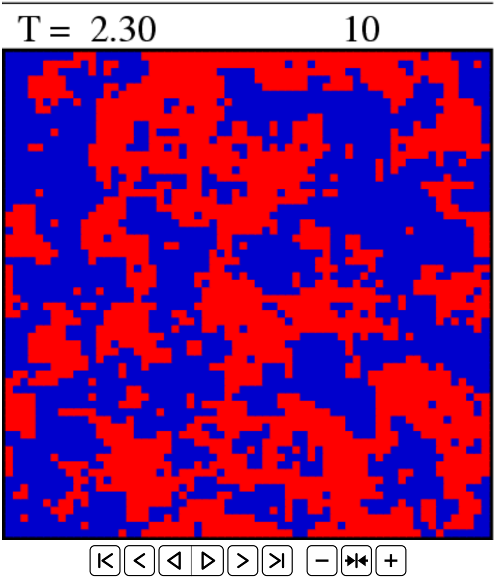
  当$M \to \infty$, 任一给定位形Γ出现的**频率**$\frac{N(\Gamma)}{M} = \frac{e^{-E(\Gamma)/k_B T}}{Z}$.

- 实现按位形的**正则分布概率**来抽取位形，而不是在相空间等概率地抽取位形

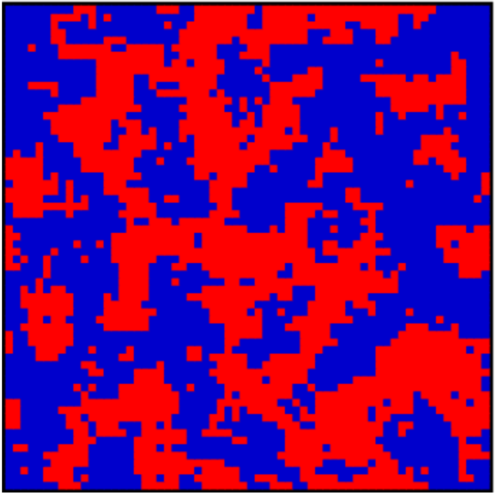
$$\boxed{\langle A \rangle \approx A_M = \sum_{\Gamma} \frac{N(\Gamma)}{M} A(\Gamma) = \frac{1}{M} \sum_{l}^{M} A(\Gamma_l)}$$

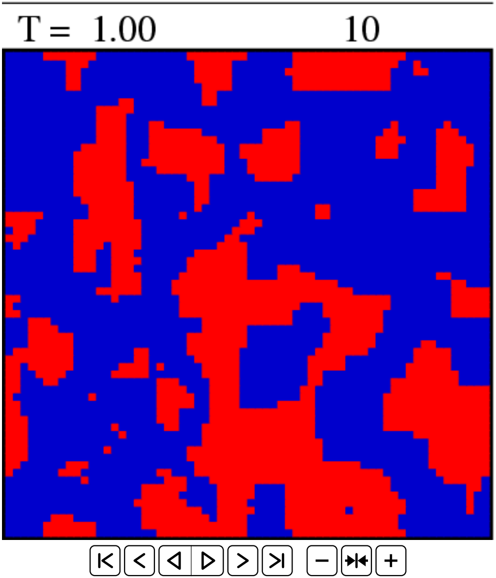
show




T = 2.30
10

T = 2.00 10




T = 1.00
10

理解相变: 序参量

- 如果外场为零，严格按配分函数计算m, 对称性要求m永远为零，因为

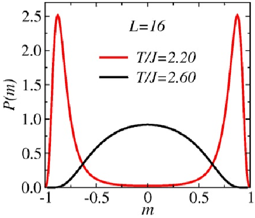
$$m = \frac{1}{N}\langle M\rangle = \frac{1}{N}\sum_{\Gamma} M(\Gamma)p(\Gamma)$$

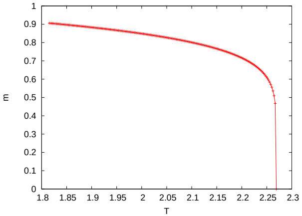
对任意一个$\Gamma = (s_1, \cdots, s_N)$, 都有唯一一个$\Gamma' = (-s_1, \cdots, -s_N)$,

$$M(\Gamma) = -M(\Gamma'), \quad p(\Gamma) = p(\Gamma')$$

这称为翻转对称性(Z_2).

- $T > T_c$ 在$m = 0$附近形成单峰, $T < T_c$ 在$\pm\bar{m}$形成对称双峰





- 在热力学极限下
  - 相变破坏了这种对称性，称为对称性自发破缺
  - 破缺的程度由m来度量，称为序参量

关联函数

- $T \gg T_c$ 热涨落 $\langle s(r)\rangle = \langle s\rangle = 0$,
- $T \to 0$, 对称破缺

$$\langle s(r)s(0)\rangle \approx \langle s\rangle^2 \approx 1^2, \qquad \text{平移不变}\to m = \langle s(r)\rangle = \langle s\rangle\text{是序参量}$$

- 定义涨落关联函数$G(r) \equiv \langle(s(r) - \langle s\rangle)(s(0) - \langle s\rangle)\rangle = \langle s(r)s(0)\rangle - \langle s\rangle^2$

$$G(r) \propto \frac{e^{-r/\xi}}{r^{d-2+\eta}}$$

- $T \to T_c$,

$$\xi \propto t^{-\nu} \to \infty,$$

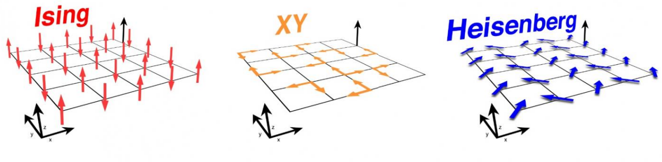
ν, η是两个最重要的临界指数, 通过标度关系决定所有指数

磁化率与序参量的涨落关系

$$\chi \propto \langle s^2\rangle - \langle s\rangle^2 \propto \int G(r) d^d r \propto \xi^{d-\eta} \propto |t|^{-\nu(d-\eta)}$$

$G(r)$的代数衰减, 或者说，ξ的发散，导致χ在T_c发散！

其他模型



$$H = -J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

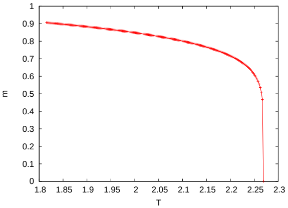
相变分别破缺不同的对称性

- Z_2 对称性
- $O(2)$ 对称性
- $O(3)$ 对称性

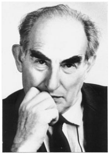
热致相变（经典相变）有没有一般性的理解?

- 由热涨落驱动, 在临界点关联长度发散， 导致奇异性

- 朗道: 利用序参量来区分具有不同的对称性的相



2D Ising transition



- 粗粒化的连续场论: LGW Hamiltonian

$$H(\mathbf{\Phi}) = \int \mathrm{d}V((\nabla\mathbf{\Phi})^2 + \frac{1}{2}s(T)\mathbf{\Phi}^2 + \frac{1}{4}u(T)(\mathbf{\Phi}^2)^2)$$

$$\mathcal{Z} = \int \mathcal{D}\mathbf{\Phi}(x)\ \ \mathrm{e}^{-H(\mathbf{\Phi})}$$

其中$\mathbf{\Phi}(\mathbf{x})$ 是局域序参量, 粗粒化场$\mathbf{\Phi}(\mathbf{x}) \sim \sum_{i\in V}\mathbf{S}_i/V$
s, u functions of T.

  - $s \sim (T - T_c)$
  - $T < T_c$长程序$m = \langle\mathbf{\Phi}\rangle \neq 0$: 对称性自发破缺

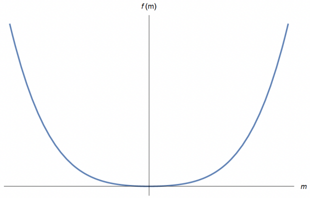
平均场理论

- 忽略其他可能的位形，只看最可几位形

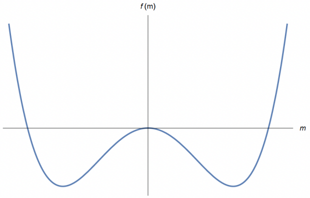
$$\frac{F}{k_BT} = -\ln Z \propto \frac{1}{2}s\mathbf{\Phi}^2 + \frac{1}{4}u\mathbf{\Phi}^4 + \cdots$$

- Landau假设： $s = s_0(T - T_c), u > 0$, 求使F极小的$\mathbf{\Phi}$.
  - $T \geq T_c, \mathbf{\Phi} = 0$
  - $T < T_c, \mathbf{\Phi} = \sqrt{-s_0(T - T_c)/u}$

对单轴铁磁系统， $\mathbf{\Phi} = m$



$T > T_c$



$T < T_c$

平均场理论

还可以计算临界指数:

- 比热不连续，但是不发散，$c \propto (T - T_c)^{-\alpha}$，$\alpha = 0$
- 磁化强度：$m \propto (T - T_c)^{1/2}$，$\beta = 1/2$
- 磁化率：$\chi \propto (T - T_c)^{-1}$，$\gamma = 1$
- 状态方程：$h \propto m^3$，$\delta = 3$

致命的弱点:

- 与空间维数无关
- 所有问题临界指数一样

过于普适了！

相变的重整化群理论

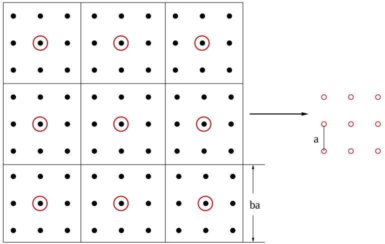
Wilson 计算机很好，但数学天分差点儿
能不能不直接求解配分函数，而找到奇异性的起源?

Wilson 发展了重整化群(Renormalization Group)理论

- 普适类: 临界指数由相互作用的对称性和空间维数决定

  例如：2D Ising, 3D Ising, 3D O(2), 3D O(3)

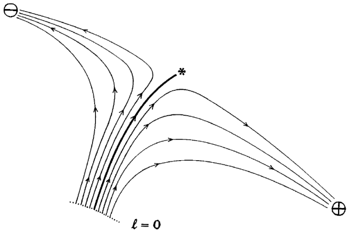
RG 变换: 保证微观尺度a不变的前提下，收放系统



标度因子$b=3$

- $\xi' = \xi/b$, 临界点关联长度无穷大: $\xi \propto (T-T_c)^{-\nu}$, 因此对应变换的不动点

$$(t', h', u') = R_b(t, h, u) \rightarrow (t^*, h^*, u^*)$$



- 步骤
  $t \propto (T-T_c), h \propto$ 外场

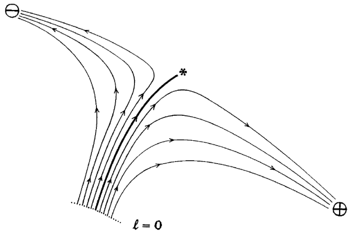
$$H(t,h,u) \xrightarrow[\text{块自旋}]{\text{粗粒化}} \bar{H}(\tilde{t},\tilde{h},\tilde{u}) \xrightarrow{rescale} H'(t',h',u')$$

- 不稳定方向决定临界指数（一般有两个）：ν, η
- 流向不动点的方向称为非关涉场(irrelevant)

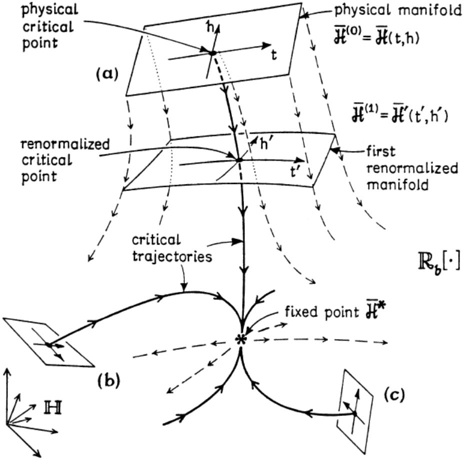
- 总自由能不变, 密度增加了b^d倍

$$f(t,h,u) = b^{-d} f(t',h',u'),$$

RG flow 和普适类



- 不稳定方向决定临界指数（一般有两个）：ν, η (y_t, y_h)
- 不同的物理模型可以流向同一个不动点
- 一个不动点决定一个普适类！



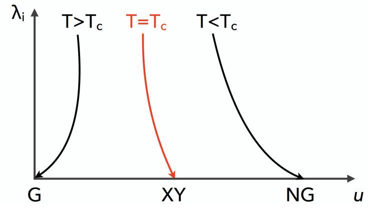
Fisher, RMP, 1998

$$\boxed{t' = tb^{y_t}, h' = hb^{y_h}, u' = ub^{y_u}}$$

3D XY model and 3D q-state clock model

自旋在二维平面内，哈密顿O(2)旋转不变

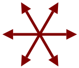
$$H = -\sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j = -\sum_{\langle i,j\rangle} \cos(\theta_i - \theta_j)$$



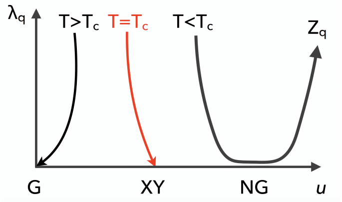
Three fixed points

- $T > T_c$ Gaussian fixed point
- $T = T_c$ XY fixed point
- $T < T_c$ XY low T fixed point (Nambu-Goldstone)

自旋在二维平面内，哈密顿Z_q旋转不变



$$H = -\sum_{\langle i,j\rangle} \cos(\theta_i - \theta_j) - h\sum_{i} \cos(q\theta_i)$$

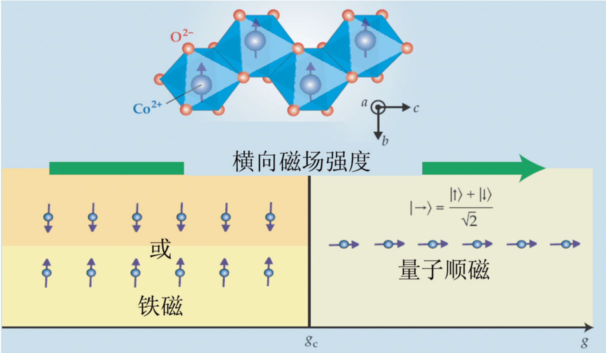


- $T > T_c$ Gaussian fixed point
- $T = T_c$ XY fixed point, O(2)对称性涌现
- $T < T_c$ Z_q low T fixed point
- λ_q 称为dangerously irrelevant field: at Z_q fixed point, $\lambda_q \neq 0$

提纲

- 相变基础知识
- 相变的统计物理描述
- 临界现象模型与理论
  - Ising model, 关联函数
  - 数值模拟: 蒙特卡洛方法
  - 序参量与关联函数
  - LGW 连续场论描述
  - 标度理论与重整化群
- 量子临界现象
  - 量子相变与模型
  - 经典量子映射
  - 场论描述与有限尺寸标度
- 退禁闭量子临界性介绍
  - 拓扑相变和拓扑激发
  - 理论介绍
  - 微观模型及其数值研究
- Spin-liquid-like state in disordered 2D quantum magnets

量子相变

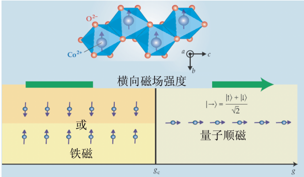


材料$CONb_2O_6$： Co^{2+}离子的自旋取向只能与晶体场轴向平行或反平行

量子相变

量子横场Ising模型

$$H = -J\sum_{<i,j>} s_i^z s_j^z - h\sum_k s_k^x \qquad J > 0$$



与经典情况不同，我们考虑$T = 0$

- $h << J$, 基态$|0\rangle = \prod_i |z,+\rangle_i$或者$\prod_i |z,-\rangle_i$

$$\lim_{|i-j|\to\infty} \langle s_i^z s_j^z \rangle \approx 1$$

- $h >> J$, 基态$|0\rangle = \prod_i |x,+\rangle_i$ 注意s^z, s^x 不对易，导致量子涨落!

$$\langle s_i^z s_j^z \rangle \propto \exp\left(\frac{-|i-j|}{\xi}\right)$$

- $h \to h_c$, 临界!

$$\xi \sim (h-h_c)^{-\nu} \to \infty, \quad \langle s_i^z s_j^z \rangle \propto \frac{1}{|i-j|^\eta}$$

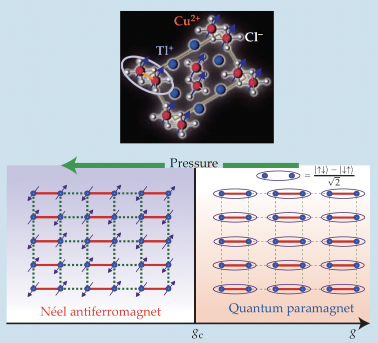
- 在h_c同样导致磁化率的发散!

另外一个量子相变的例子

- 反铁磁Néel-顺磁量子相变: demerized AF Heisenberg model

$$H = \sum_{<i,j>} J_{ij} S_i \cdot S_j$$

$J_{ij} = J_2, J_1$: $g = J_2/J_1$



- $g < g_c$, 反铁磁Néel 序
- $g > g_c$, dimer 相;
  dimer 波函数是$(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$的直积态，量子顺磁.

Quantum rotor model

- 每个格点上有一个转子(rotor): 粒子在$N-1$ 维球面上运动. $\mathbf{n}$是它的位置算符，$\mathbf{L}$是它的角动量算符。
- Hamiltonian

$$H_R = \frac{Jg}{2}\sum_i \mathbf{L}_i^2 - J\sum_{\langle ij\rangle}\mathbf{n}_i\cdot\mathbf{n}_j = \int dx \left(\frac{Jg\mathbf{L}^2}{2} + \frac{J}{2}(\nabla\mathbf{n})^2\right)$$

  $1/(Jg)$转动惯量, J 耦合作用
- $g \gg 1$, 转动动能主导，量子顺磁

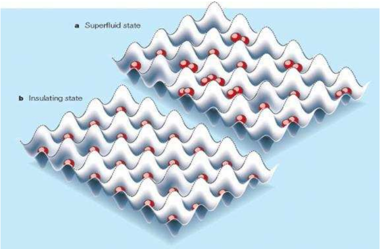
$$\langle 0|\mathbf{n}_i\cdot\mathbf{n}_j|0\rangle \sim e^{-|x_i-x_j|/\xi}.$$

- $g \ll 1$, 耦合主导，磁有序

$$\lim_{|x_i-x_j|\to\infty}\langle\mathbf{n}_i\cdot\mathbf{n}_j\rangle \approx 1$$

Bose-Hubbard 模型

Bose-Hubbard 模型可以描述光晶格中的玻色冷原子

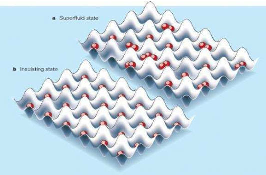


$$H = -t \sum_{<ij>} a_i^+ a_j + \frac{U}{2} \sum_i n_i(n_i - 1) - \mu \sum_i n_i$$

- t 反映了粒子在近邻格点间的隧穿能量
- U 描述单个格点上的原子间的相互作用(我们只研究 $U > 0$)
- μ 是化学势

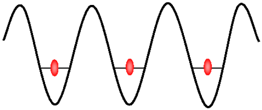
基态

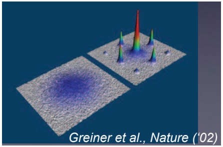
1. $t \gg U$， 粒子的位置完全不确定，系统处于超流相(SF)



$$|\psi_{SF}\rangle_{U=0} \propto (\sum_{i=1}^{M} a_i^{+})^N |0\rangle$$

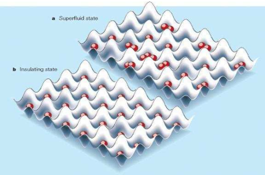
2. $t \ll U$， 等量填充时，每个格点占据相同数目的原子使总能量取最小，系统处于 Mott 绝缘体相(MI)

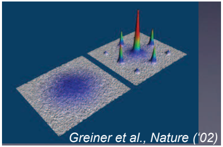


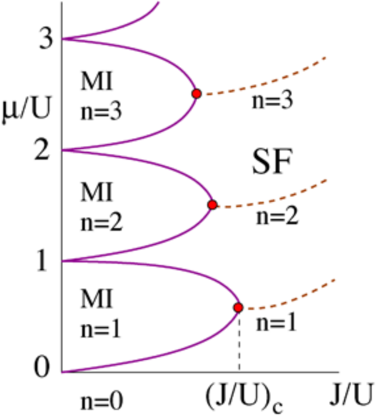


$$|\psi_{MI}\rangle_{t=0} \propto \prod_{i=1}^{M} (a_i^{+})^n |0\rangle$$

相图







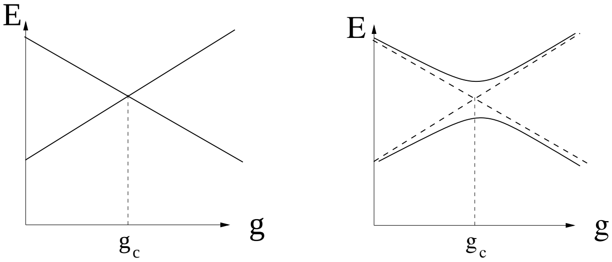
- MI: integer filling, insulating, gaped
- SF: any filling fraction, gapless

量子相变的一般描述

考虑一个Hamiltonian

$$H(g) = H_0 + gH_1$$

- $[H_0, H_1] = 0$: 同时对角化，激发态与基态交叉于g_c:一级相变



- $[H_0, H_1] \neq 0$: 当$L \to \infty$，两个能级在g_c无限靠近，基态对称性发生变化

- 关联长度发散

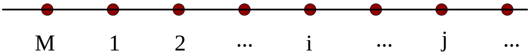
$$\xi \sim (g - g_c)^{-\nu},$$

- 能隙(或特征能量涨落) Δ 定义动力学临界指数z

$$\Delta \sim (g - g_c)^{z\nu} \sim \xi^{-z}, 1/\Delta = \xi_\tau$$

经典与量子的映射

考虑一条经典Ising 链, 无外场($B = 0$)



其配分函数可以通过转移矩阵计算

$$\mathcal{Z} = \sum_{\{s_i\}} \mathrm{e}^{-H/T} = \sum_{\{s_i\}} \prod_{i=1,m} T_1(s_i, s_{i+1})$$

其中$T_1(s_1, s_2) = \mathrm{e}^{Ks_1s_2}$,, 其中$K = J/T$
将其理解为矩阵元

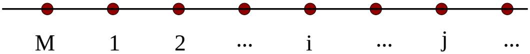
$$\mathcal{Z} = \mathrm{Tr}(T_1^M)$$

其中

$$T_1 = \begin{pmatrix} \mathrm{e}^{K} & \mathrm{e}^{-K} \\ \mathrm{e}^{-K} & \mathrm{e}^{K} \end{pmatrix},$$

称为转移矩阵(transfer matrix)

经典与量子的映射



- T_1的本征值为

$$\epsilon_{1,2} = \mathrm{e}^{K} \pm \mathrm{e}^{-K}$$

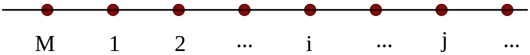
- 本征矢为

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = |x, +\rangle, \qquad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = |x, -\rangle$$

- 自由能密度

$$f = -\frac{T \ln \mathcal{Z}}{M} = -\frac{T}{M}\ln\left(\epsilon_1^M \left(1 + \frac{\epsilon_2}{\epsilon_1}\right)^M\right) \approx T\epsilon_1$$

经典与量子的映射



- 关联函数

$$\langle s_i s_j \rangle = \frac{1}{\mathcal{Z}} \sum_s \mathrm{e}^{-H/T} s_i s_j = \frac{1}{\mathcal{Z}} \mathrm{Tr}(T_1^i \sigma_z T_1^{j-i} \sigma_z T_1^{M-j})$$

When $M \to \infty$, 利用T_1本征矢为基矢, 注意$\sigma_z|x,+\rangle = |x,-\rangle$

$$\langle s_i s_j \rangle = (\frac{\epsilon_2}{\epsilon_1})^{j-i} = (\tanh K)^{j-i}$$

关联长度ξ

$$\mathrm{e}^{-\frac{j-i}{\xi}} = (\tanh K)^{j-i}, \qquad \frac{1}{\xi} = -\ln(\tanh K)$$

当$T \to 0, K \to \infty$, 有$\xi \to \infty$. 可认为$T_c = 0$.

经典与量子的映射

我们改写

$$T_1 = e^K \begin{pmatrix} 1 & e^{-2K} \\ e^{-2K} & 1 \end{pmatrix} = e^K(1 + e^{-2K}\sigma_x)$$
$$\approx e^K(1 + \frac{1}{2\xi}\sigma_x) \approx e^{K + \frac{1}{2\xi}\sigma_x}$$

在$\xi \to \infty$的极限下,

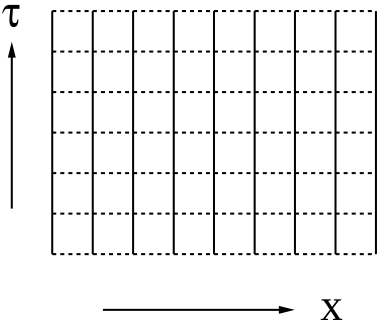
$$T_1 = e^{-\mathcal{H}_Q}$$

其中$\mathcal{H}_Q = -K - \frac{1}{2\xi}\sigma_x$

$$\boxed{\mathcal{Z} = \mathrm{Tr}(T_1)^M = \mathrm{Tr}\, e^{-M\mathcal{H}_Q} = \mathrm{Tr}\, e^{-\frac{\mathcal{H}_Q}{1/M}}}$$

- 这是一个温度$T = 1/M \to 0$的量子系统!
- 一维无限长经典Ising链等价于零温下一个磁场中的量子自旋!
- $1/\xi = -\ln(\tanh J/T)$对应横向磁场.
- 经典系统‘相变’温度为$T = 0$, 零温量子系统‘相变’磁场为$1/\xi$
- 能隙$\Delta = 1/\xi$

$(D+1)$ 维经典$\Leftrightarrow D$ 维量子映射: 转移矩阵



- 任意挑选一个方向作为(虚)时间方向, 构造转移矩阵T.

$$\mathcal{Z} = \mathrm{Tr}\; T^M$$

- 转移矩阵把系统状态从一行转移到另一行, 相当于

$$T = \mathrm{e}^{-\mathcal{H}_Q}$$

- τ方向相互作用贡献非对角项，比如‘横场’

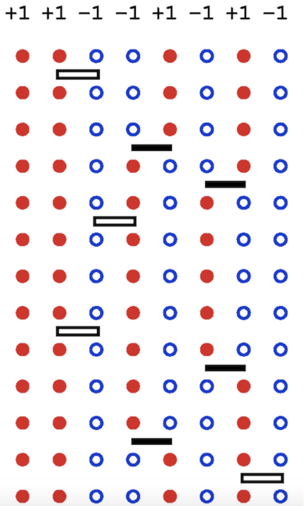
- 对应一个量子系统的配分函数

$$\mathcal{Z} = \mathrm{Tr}\; e^{-M\mathcal{H}_Q} = \mathrm{Tr}\; e^{-\beta\mathcal{H}_Q}$$

- 量子系统的倒温度$\beta = M \to \infty$
  - 二维经典Ising模型$\to$ 一维量子横场Ising模型, $J/T \to J/h$
  - 二维经典Heisenberg 模型$\to$ 一维量子rotor 模型($N = 3$)

量子蒙卡可以理解为寻找量子模型的经典对应

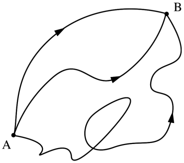
- An SSE configuration



$$\langle A\rangle = \frac{\mathrm{Tr}\{Ae^{-\beta H}\}}{\mathrm{Tr}\, e^{-\beta H}} \rightarrow \frac{\sum_c A_c W_c}{\sum_c W_c}$$

- S_z basis
- diagonal and loop updates
- observables and estimators
  - energy estimator : number of operators, $H_c = -n/\beta$
  - spin stiffness estimator : winding number fluctuations
    $$\rho_s = \frac{\langle W_\alpha^2\rangle}{L^{d-2}\beta}$$
  - staggered magnetization $m_{sz} = \sum_i (-1)^{i_x+i_y} s_{iz}/N$
- 缺陷：有些量子模型很难找到非负权重的经典表示→ 符号问题

量子相变的场论描述



- 粗粒化场$\mathbf{\Phi}(\mathbf{r},\tau) \sim \sum_i \mathbf{s}_i(\tau)/\Delta V$
- 时间可以‘转动’为温度
- 通过路径积分 把D维量子系统 映射到$D+1$维经典系统, 得到Landau-Ginzburg-Wilson形式的作用量

$$\mathcal{S}(\mathbf{\Phi}) = \int \mathrm{d}V\mathrm{d}\tau (v^2(\partial_\tau \mathbf{\Phi})^2 + (\nabla_x \mathbf{\Phi})^2 + \frac{1}{2}s\mathbf{\Phi}^2 + \frac{1}{4}u(\mathbf{\Phi}^2)^2)$$

$$Z = \int \mathcal{D}\mathbf{\Phi}(x,\tau)\ \mathrm{e}^{-\mathcal{S}(\mathbf{\Phi})}$$

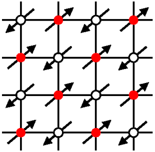
- 量子涨落
    - Wilson注意到统计物理与量子场论的深刻联系

    - 一般的量子相变都可以用通常的Landau-Ginzburg-Wilson 理论来描述

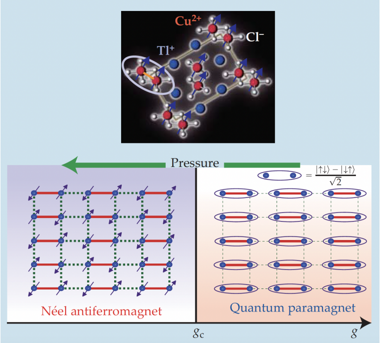
时空维度$D+1$ 和对称性决定普适类

AF Néel-Paramagnetic 量子相变

- 二维热力学极限，基态破缺O(3)对称性, Néel态



$$\langle \mathbf{m}_s \rangle = \langle \frac{1}{N} \sum_i \mathbf{S}_i (-1)^{x_i + y_i} \rangle \neq 0$$

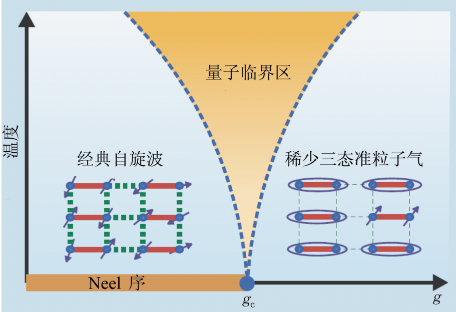


- ▶ 普适类：3D O(3), 已被量子蒙特卡洛模拟证实, LGW 成功！
- ▶ 实验实现

What's special about quantum criticality?

- large $T > 0$ quantum critical "fan" where T is the only relevant energy scale

- 物理量遵从由$T = 0$临界点决定的power laws



- 改变T等效于改变虚时尺寸L_τ, 利用有限尺寸标度分析可以得到如下power laws

$$\xi \propto T^{-1} \quad \text{关联长度}$$

$$C \propto T^2 \quad \text{比热}$$

$$\chi(0) \propto T \quad \text{uniform 磁化率}$$

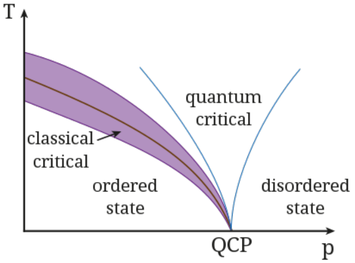
量子系统的‘经典’相变

一个量子系统，固定参数，比如$J, g,$改变温度，会发生什么?

量子系统的‘经典’相变

一个量子系统，固定参数，比如J, g, 改变温度，会发生什么？

升高温度后，量子涨落在‘宏观小，微观大’的所谓‘介观’尺度内被平均掉: 通常的热涨落驱动的相变，也就是‘经典’相变



- 二维量子Ising 模型
  如果$h = 0$, 在零温下沿z方向自发磁化（有序）。随着温度升高，也会在居里点变成顺磁态。
  这个相变就是普通的Ising模型相变！
- 在$T = 0$，改变横场h，发生的是**量子相变,**
  等价于一个**3维经典Ising 模型的热相变**！

提纲

- 相变基础知识
- 相变的统计物理描述
- 临界现象模型与理论
  - Ising model, 关联函数
  - 数值模拟: 蒙特卡洛方法
  - 序参量与关联函数
  - LGW 连续场论描述
  - 标度理论与重整化群
- 量子临界现象
  - 量子相变与模型
  - 经典量子映射
  - 场论描述与有限尺寸标度
- 退禁闭量子临界性介绍
  - 拓扑相变和拓扑激发
  - 理论介绍
  - 微观模型及其数值研究
- Spin-liquid-like state in disordered 2D quantum magnets

是不是所有的量子相变都可以纳入朗道-金兹堡-威尔逊范式?

是不是所有的量子相变都可以纳入朗道-金兹堡-威尔逊范式?

事情没有这么简单
或者说比这有趣!

KILORAN HOWARD BROWN UNIVERSITY DENISE APPLEWHITE/PRINCETON UNIV.

Thouless Kosterlitz Haldane

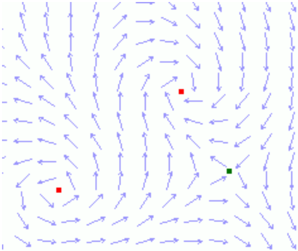
这三位2016年诺贝尔奖获得者将拓扑引入了物理学

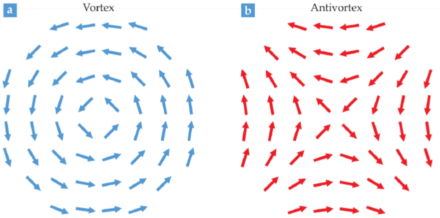
拓扑相变

$$H = -\sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j = -\sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j)$$

没有长程序的二维*XY*类模型中发生的相变:

Kosterltiz-Thouless相变





涡旋拓扑荷

$$q = \frac{1}{2\pi} \oint \nabla \theta_{vor} \cdot d\vec{l}$$

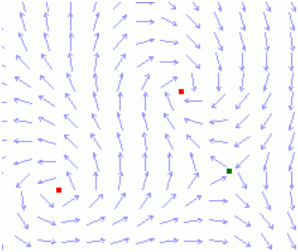
- 存在拓扑激发(或缺陷): 涡旋

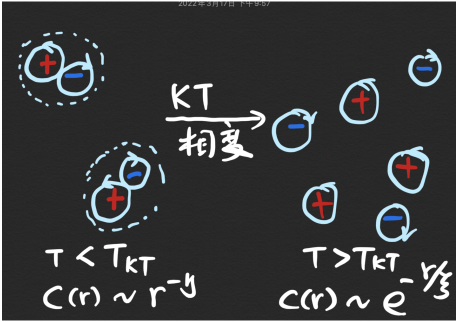
拓扑相变

$$H=-\sum_{\langle i,j\rangle}\vec{S}_i\cdot\vec{S}_j=-\sum_{\langle i,j\rangle}\cos(\theta_i-\theta_j)$$

没有长程序的二维XY类模型中发生的相变:

Kosterltiz-Thouless相变





- 正负涡旋在$T<T_c$时束缚在一起
- $T>T_c$涡旋对解束缚导致相变
- 不能用局域序参量描述

Haldane 的推广

一维量子反铁磁Heisenberg 模型

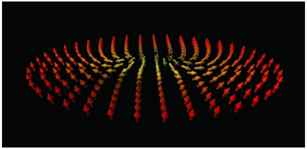
$$H_0 = J\sum \mathbf{S}_i \cdot \mathbf{S}_j$$

受KT思想启发，Haldane注意到了路径积分中的拓扑

$$\mathcal{S}_n = \frac{1}{2g}\int d\tau \int dx \left[\frac{1}{c^2}(\frac{\partial \mathbf{n}}{\partial \tau})^2 + (\frac{\partial \mathbf{n}}{\partial x})^2\right] + S_B; \qquad \mathcal{Z} = \int \mathcal{D}\mathbf{n}\ \mathrm{e}^{-S_n}$$

$\mathbf{n} = (-1)^x \mathbf{S}_i$; S_B是拓扑项, 来自量子力学Berry phase, 带来 干涉效应

- 拓扑缺陷skyrmion
- skyrmion number Q类似于涡旋的拓扑荷, $S_B = i2\pi SQ$
- 自旋整数的系统$\exp(S_B) = 1$; Haldane phase, 拓扑导致的边缘态
- 自旋半整数系统$\exp(S_B) = (-1)^Q$; 临界态$G(R) \propto r^{-\eta}$



$$Q = \frac{1}{4\pi}\int d\mathbf{x}\ \ \mathbf{n}\cdot(\partial_1 \mathbf{n} \times \partial_2 \mathbf{n})$$

二维系统?

- 在AF Néel序, 由于序参量平滑变化，相邻两条链的Berry phase 抵消，即使$S = 1/2$系统S_B也不起作用

  Chakravarty, Halperin, Nelson, PRB 39, 2344(1989)

- 如果考虑引入竞争项, 并导致离开反铁磁相的相变, 情况就不同了

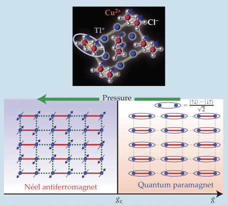
$$H = J \sum \mathbf{S}_i \cdot \mathbf{S}_j + gH_1$$

- Demerized Heisenberg 模型每个原胞中有偶数个$S = 1/2$自旋,

  它的无序态是平庸的顺磁态，拓扑激发在其中不起作用，只是破坏长程序

  可以用LGW理论描述，属于3D O(3)普适类

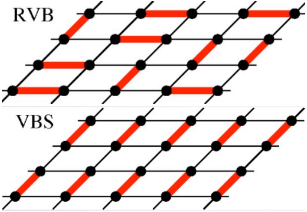
- 更有趣的非磁性基态；每个原胞有一个$S = 1/2$自旋



非磁性而非平庸基态

- $\boxed{\langle \mathbf{m}_s \rangle = \langle \frac{1}{N} \sum_i \mathbf{S}_i (-1)^{x_i+y_i} \rangle = 0}$

  ▶ Valence bond(价键态或自旋单态) 

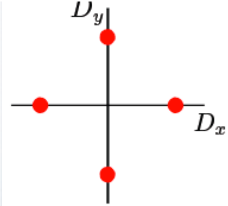


- resonating valence-bond (RVB) 自旋液体(spin liquid)
  没有任何长程序

- valence-bond solid (价键固体VBS)
  破缺了晶格的平移和旋转对称性

VBS 序参量(D_x, D_y)

$$\boxed{D_x = \frac{1}{N} \sum_{i=1}^{N} (-1)^{x_i} \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{x}},\ D_y = \frac{1}{N} \sum_{i=1}^{N} (-1)^{y_i} \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{y}}}$$



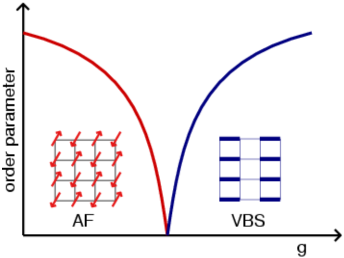
$$\langle (D_x, D_y) \rangle \neq 0$$

VBS由拓扑激发导致 Read and Sachdev, PRL 62, 1694(1989)

- 从Néel到VBS的相变是怎样的?

Deconfined quantum criticality 解禁闭量子相变

从Néel-VBS的连续相变: 与LGW理论不同 Senthil et al Science (2004)



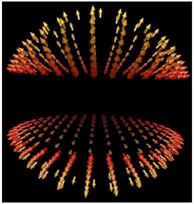
- 顺磁相quadrupled monopoles 导致VBS: relevant
  Read and Sachdev,PRL 1989
- 相变点处拓扑激发不起作用：irrelevant,
  3D defect-suppressed O(3)普适类
- 后果很严重：violate Landau's rule;
  既不是3D O(3) 也不是3D O(2); 3D Z_4模型中Z_4 各向异性是dangerously irrelevant

利用spinon场表示的场论

- 两边的序参量都由自旋1/2的spinon场得到
- $NCCP^1$ action

$$\mathcal{S}_z = \int \mathrm{d}r^2 \mathrm{d}\tau [|(\nabla - iA)z|^2 + s|z|^2 + u(|z|^2)^2) + \kappa(\nabla \times A)^2]$$

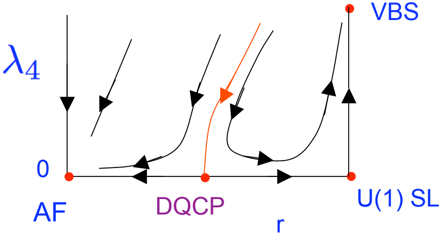
- VBS一侧spinon感受到线性势:confined, 在临界点deconfine



Q的改变对应monopole

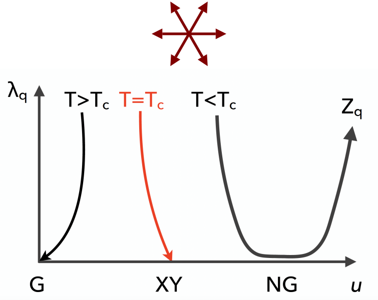
Expected RG flows in DQC

λ_4是拓扑缺陷的逸度(fugacity)



from M.Fisher

Z_q 模型的重整化流图



Skyrmion的dangerously irrelevant行为类似于3D Z_4 models:

Z_4 各向异性的dangerously irrelevant

- correlation length $\xi \propto (g - g_c)^{-\nu}$;
- cross-over length scale from XY order to Z_q order $\xi' \propto (g - g_c)^{-\nu'}$

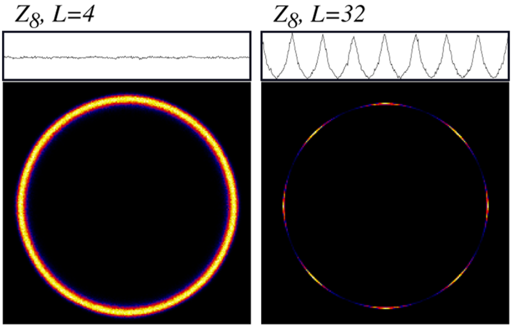
两尺度临界行为！

通过MC 模拟显示Z_q 模型的crossover 行为

- standard order parameter (m_x, m_y)

$$m_x = \frac{1}{N}\sum_i \cos(\theta_i), \qquad m_y = \frac{1}{N}\sum_i \sin(\theta_i)$$

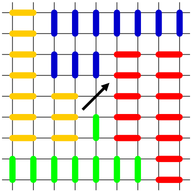
- probability distribution $P(m_x, m_y)$ shows cross-over from U(1) to Z_q for $T < T_c$

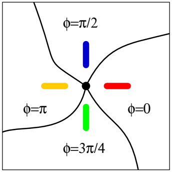


Lou, Balents, Sandvik, PRL 2007

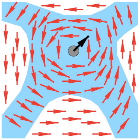
在同一个温度下，不同尺寸的行为

Deconfined Quantum Criticality, picture from VBS side

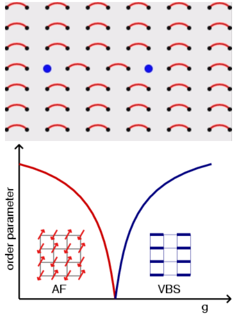




- Topological defect: Z_4 vortex form at nexus of four domain walls



- ξ' is the thickness of domain wall, diverges faster than ξ
- At the core, there's an unpaired spin $\rightarrow$ spinon
  different from Z_4 vortex of the $q = 4$ clock model



- Spinons bind together in the VBS state (confinement) and condensate in the Néel state, deconfine at the critical point
- leading to a continuous phase transition in a New universality

Levin and Senthil, PRB 70, 2004

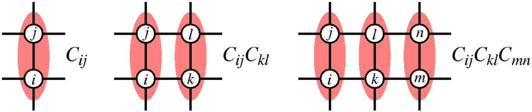
numerical study of the DQC

数值模拟方法研究DQC–JQ model

海森堡模型加上多自旋相互作用: ”人造”J-Q 模型

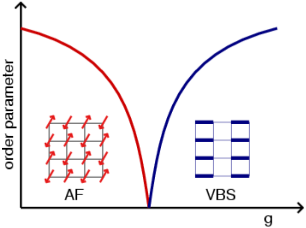
Sandvik, PRL, 2007

$$H = -J\sum_{\langle ij\rangle} C_{ij} - Q\sum_{\langle ijkl\rangle} C_{ij}C_{kl}, \quad C_{ij} = (\frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_j)$$

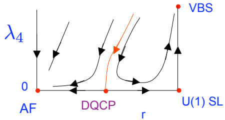


Lattice symmetries are kept

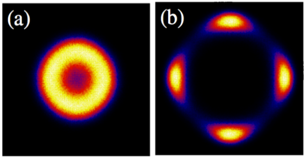
- large Q, columnar VBS
- small Q, Néel
- No sign problem
- ideal for QMC study of the DQC physics



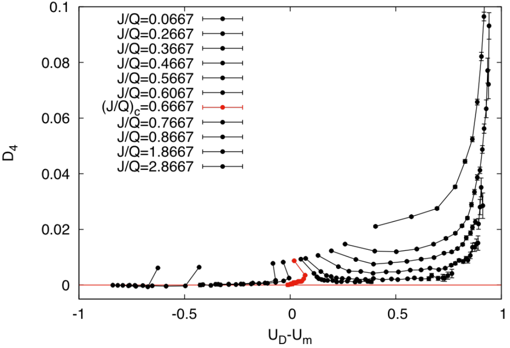
Emergent U(1) and RG flows in the J-Q_3 model



$P(D_x, D_y)$: emergent U(1) symm



- Define D_4 as m_q in the Z_q model to quantify Z_q anisotropy
- Use Binder cumulants U_m and U_D
  - $U_m - U_D$ shows flows to dqc, Neel and VBS



Shao, Guo, Sandvik (work in progress)

- Shows similarity with the Z_q models, but Z_q models only have one ordered phase, different universality expected

标度失效(scaling violation)

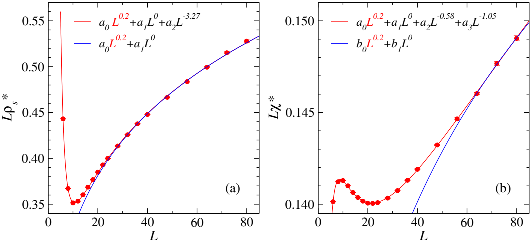
自旋劲度$\rho_s \propto \delta^{\nu(d+z-2)}$, 磁化率$\chi \propto \delta^{(d-z)\nu}$

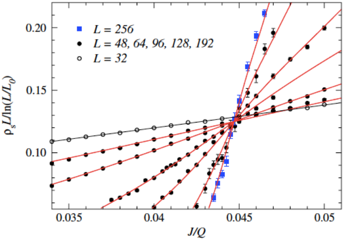
Conventional FSS

$$\boxed{\rho_s(\delta, L) = L^{-\nu(d+z-2)/\nu} f(\delta L^{1/\nu}), \qquad \chi(\delta, L) = L^{-\nu(d-z)/\nu} f(\delta L^{1/\nu})}$$

At critical point: $\rho_s \propto L^{-(d+z-2)} = L^{-z}, \quad \chi \propto L^{-(d-z)}$

$z = 1$ for J-Q model, $\rho_s L$ and χL should be constants at q_c





- $z \neq 1$ **does not work**
- **large scaling corrections?** Sandvik PRL 2010, Bartosch PRB 2013
- **weak first-order transition?** Chen et al PRL 2013 这意味着DQC理论的失败困境！ **Nahum et al PRX, 2015; arXiv: 1506.06798**

量子临界两尺度标度

Shao, Guo, and Sandvik, Science 2016

- 两个发散长度由一个参数控制$\xi \propto \delta^{-\nu}$， $\xi' \propto \delta^{-\nu'}$
- 某个物理量A的有限尺寸标度，在热力学极限下$A \propto \delta^{\kappa}$
- Conventional scenario

$$A(\delta, L) = L^{-\kappa/\nu} f(\delta L^{1/\nu}, \delta L^{1/\nu'}), \quad \boxed{A(\delta = 0, L) \propto L^{-\kappa/\nu}}$$

$L \to \infty, f(\delta L^{1/\nu}, \delta L^{1/\nu'}) \to (\delta L^{1/\nu})^{\kappa}$, recovers $A \propto \delta^{\kappa}$

- We propose

$$A(\delta, L) = L^{-\kappa/\nu'} f(\delta L^{1/\nu}, \delta L^{1/\nu'}), \quad \boxed{A(\delta = 0, L) \propto L^{-\kappa/\nu'}}$$

when $L \to \infty, f(\delta L^{1/\nu}, \delta L^{1/\nu'}) \to (\delta L^{1/\nu'})^{\kappa}$ leads to $A \propto \delta^{\kappa}$

For example: spin stiffness ρ_s, $\kappa = \nu(d + z - 2) = 1$. At q_c

$$\boxed{\text{NOT } \rho_s \propto L^{-1}, \quad \text{BUT} \quad \rho_s \propto L^{-\nu/\nu'}}$$

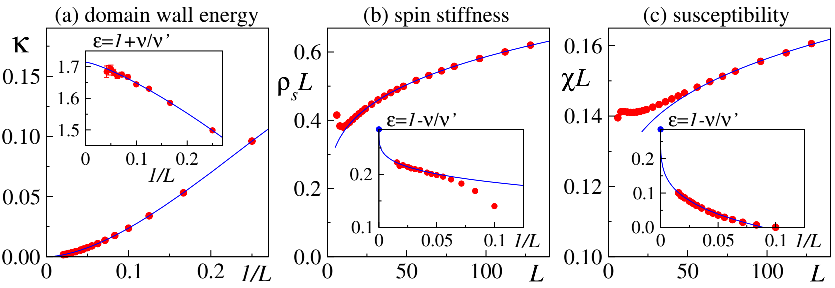
- We provide a phenomenological explanation

Evidence for unconventional scaling

Shao, Guo, and Sandvik, Science 2016
according to our scaling form

$$\rho_s \sim L^{-\nu/\nu'}, \quad \text{instead of } \rho_s \sim L^{-1}$$

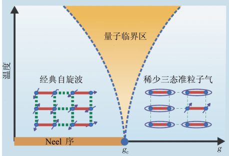
$$\chi \sim L^{-\nu/\nu'}, \quad \text{instead of } \chi \sim L^{-1}$$



- This explains drifts in $L\rho_s$ and χL in J-Q and other models ($z = 1, d = 2$)

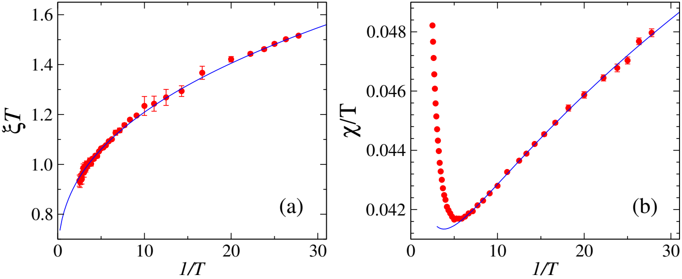
有限温度下的反常标度行为

- $\beta = 1/T$ 也是‘有限尺寸’：
  $L \to \beta^{1/z}$
- 通常的标度理论($z = 1$ for J-Q)
  - $\xi \sim L$ 给出$\xi_T \propto \beta^{1/z} = T^{-1}$,
  - $\chi \sim L^{-(d-z)}$ 给出$\chi_T \propto \beta^{-(d-z)/z} = T$



- 新的标度,考虑ν/ν':

$$\boxed{\xi_T \propto T^{-\nu'/\nu};\ \chi \sim L^{-\nu/\nu'} \text{ leads to } \chi_T \propto T^{\nu/\nu'}}$$



Shao, Guo, and Sandvik, Science 2016

结论

- 简单的两尺度标度假设可以解释“标度违背”
- 对于$T > 0$我们利用有限尺寸标度形式可以得到新的标度律

Reference:

1 Science 354, 213 (2016)

提纲

- 相变基础知识
- 相变的统计物理描述
- 临界现象模型与理论
  - Ising model, 关联函数
  - 数值模拟: 蒙特卡洛方法
  - 序参量与关联函数
  - LGW 连续场论描述
  - 标度理论与重整化群
- 量子临界现象
  - 量子相变与模型
  - 经典量子映射
  - 场论描述与有限尺寸标度
- 退禁闭量子临界性介绍
  - 拓扑相变和拓扑激发
  - 理论介绍
  - 微观模型及其数值研究
- Spin-liquid-like state in disordered 2D quantum magnets

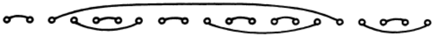
- $\boxed{\text{Searching for Spin-liquid-like state in 2D quantum magnets}}$

Random Singlet state

- Random Singlet (RS) state: No any longrange order, correlations decay algebraically, a spin-liquid-like state

- 1D: Random Singlet state in random J $S = 1/2$ chain is well known

  - Each spin is paired with one other spin that maybe very far away



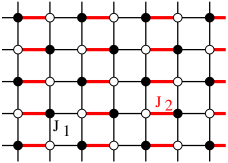
  - using SDRG, properties of the Infinite Randomness Fixed Point are found

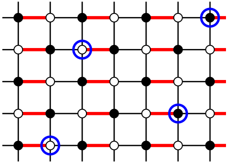
  - very slow dynamics $z \to \infty$
  - typical corr. $C^{typ}(r) \propto \exp(-cr^{1/2})$
  - but mean spin correlation $C(r) \propto 1/r^2$, dominated by rare long VBs

  Dasgupta and Ma, PRB 22, 1305(1980); D. Fisher, PRB 50, 3799 (1994)

- exist (or stable) in 2D unfrustrated systems? Controversial

Simpler system: site diluted Heisenberg dimer system





For intact system

- $J_2/J_1 > 1.91$ dimerized phase

When diluted

- Unpaired 'dangling spin' moments form in gapped host system
- Effective bipartite interactions Nagaosa, et al J.P.S.J, 1996
- Moments form weak AFM order

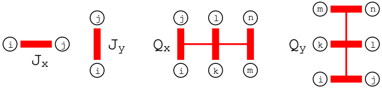
Is this the faith of the spinons in the square-lattice disordered VBS?

- Kimchi, Nahum and Senthil: Most likely yes
  frustrated interactions required to induce RS state
  PRX 2018
- Our conclusion: RS phase exists in 2D unfrustrated systems

Models, schematic phase diagrams

- Introduce randomness to the 2D J-Q_3 model

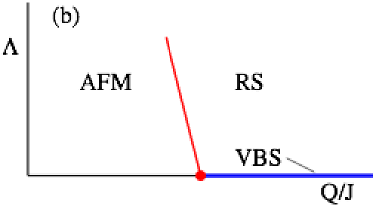
$$H = -J \sum_{\langle ij \rangle} C_{ij} - Q \sum_{\langle ijklmn \rangle} C_{ij} C_{kl} C_{mn},$$



$$C_{ij} = (\frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_j)$$

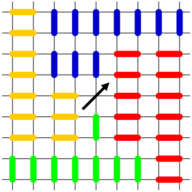
  - random Q: e.g. $Q_{ijklmn} = 0, 2Q$ randomly
  - random J: e.g. $J_{ij} = 0, 1$ or $J_{ij} \in [1 - \Delta, 1 + \Delta]$

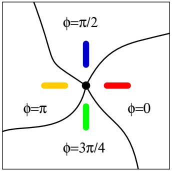
- We find the schematic phase diagrams: Λ represents disorder strength



Mechanism of RS state formation

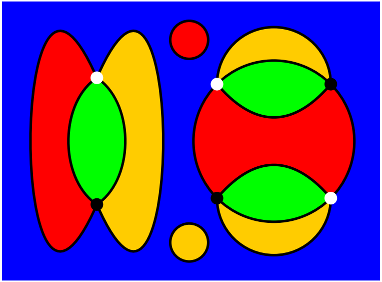
- Clean system, there's no static domain wall
- spinon pairs move in the whole space-time





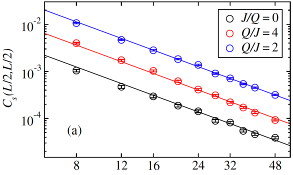
However, when disorder introduced

- According to Imry-Ma argument,
  - any amount of disorder in a VBS will cause domain formation: destroy VBS
- Spinons form in pairs (Not random distribution) and frozen:
  a pair (left) and a quadruplet (right)
- domain walls mediate spinon-spinon interactions
- pairing avoids AFM order, instead power-law correlations



Properties of the RS phase: correlations

- C_s: spin-spin correlation;



random Q model

For large Q/J:

- $C_s(L/2, L/2) \propto 1/L^2$
- Scaling: $C_s(r) \propto 1/r^{D+z-2+\eta}$

therefore,

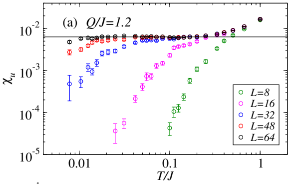
$$\boxed{z+\eta=2}$$

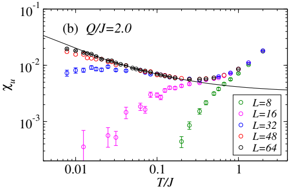
- Finite temperature behavior of the uniform susceptibility

$$\chi_u \propto T^{d/z-1}$$

we found

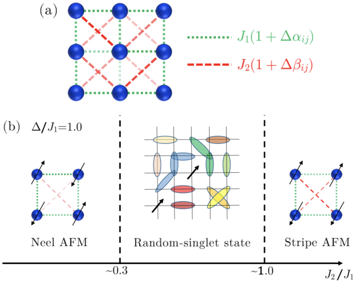
- $z=2$ at transition points
- $z>2$ inside the RS phases





Similar results found in frustrated system

H.D. Ren, T.Y. Xiong, H.Q. Wu, D.N. Sheng, and S.S. Gong, arXiv: 2004.02128

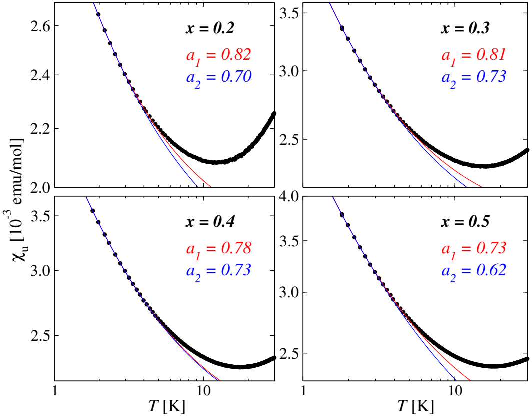


- DMRG calculations with 2D J_1-J_2 Heisenberg model, frustrated system
- confirm universal RS state with $z + \eta = 2$

RS state, Not just in JQ model, a universal behavior

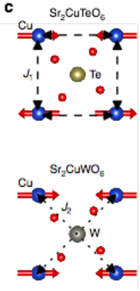
Experiments: Some 'disordered spin liquids' may actually be RS states

- Recent example $Sr_2CuTe_{1-x}W_xO_6$
  square lattice $S = 1/2$ system with J_1 or J_2 randomly on plaquettes



χ_u for $x = 0.2, 0.3, 0.4, 0.5$

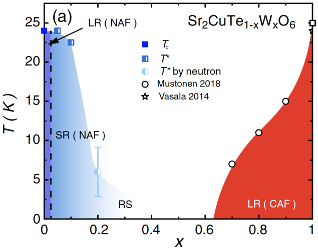
Watanabe, et al, arXiv:1808.02614

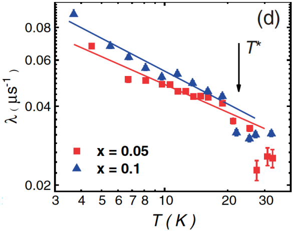


- The fitting curves are
  $\chi_u = \chi_0 + cT^{-a}$
  $a = 1 - d/z$
  - red: $T < 4K$
  - blue: $T < 3K$
- different from Curie's law

Experiments: Some 'disordered spin liquids' may actually be RS states

Experiment done by ShiLiang Li's group
Hong, Liu, et al, PRL, 126, 037201 (2021)





Time-dependent zero-field μSR spectra

- The relaxation rate exhibits quantum-critical scaling with dynamic exponent $z > 2$

$$\lambda \propto T^{-a} \qquad a = 1 - d/z.$$

We found

- for $x = 0.05$, $a = 0.35 \pm 0.03 \to z = 3.0 \pm 0.2$
- for $x = 0.1$, $a = 0.42 \pm 0.03 \to z = 3.5 \pm 0.3$

Conclusions

- We found RS phase in unfrustrated 2D system; but not infinite-randomness fixed point (z is finite)
- We can not rigorously exclude weak AFM order
  - unlikely, in light of well-characterized AFM-RS critical point
- The RS phase may be universal
  - properties can be investigated in great detail with QMC
  - comparisons with experiments possible, e.g., varying z. Some 'disordered spin liquids' may actually be RS states

References:

1. L. Liu, H.Shao, Y.Lin, WG, A. Sandvik, Phys. Rev. X **8**, 041040 (2018)
2. L. Liu, WG, A. Sandvik, Phys. Rev. B **102**, 054443(2020)
3. W. Hong, L. Liu, et al, Phys. Rev. Lett. **126**, 037201 (2021)

主要参考文献

1 S. Sachdev, Quantum Phase Transitions (Cambridge University Press, Cambridge, England, 1999).

2 A. W. Sandvik， AIP Conf. Proc. 1297, 135 (2010).